

# **Separating Physics from Chemistry in Diffuse Spectroscopy:**

Light scattering and light absorbance  
separated by  
Extended Multiplicative Signal Correction (EMSC)  
and variations thereof

**Harald Martens**

Harald.Martens@matforsk.no

NIR 2003, Cordoba Spain April 2003

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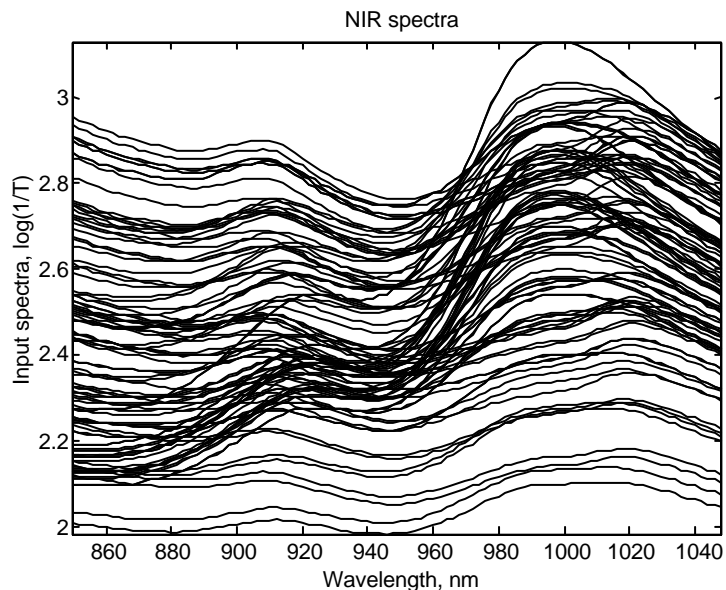
Acknowledgements:

Jesper Pram Nielsen, Michael Bom Froest,  
Morten Beck Rye, Xuxin Lai and Achim Kohler

# 100 NIR spectra of mixtures of two powder types: wheat gluten and wheat starch.

Five different mixtures (0,25,50,75 and 100% gluten)

20 replicates of each (powder sampling, cuvette filling, sample packing, different sample holders, spectral parallels)



Martens, H., Pram Nielsen, J. and Balling Engelsen, S:

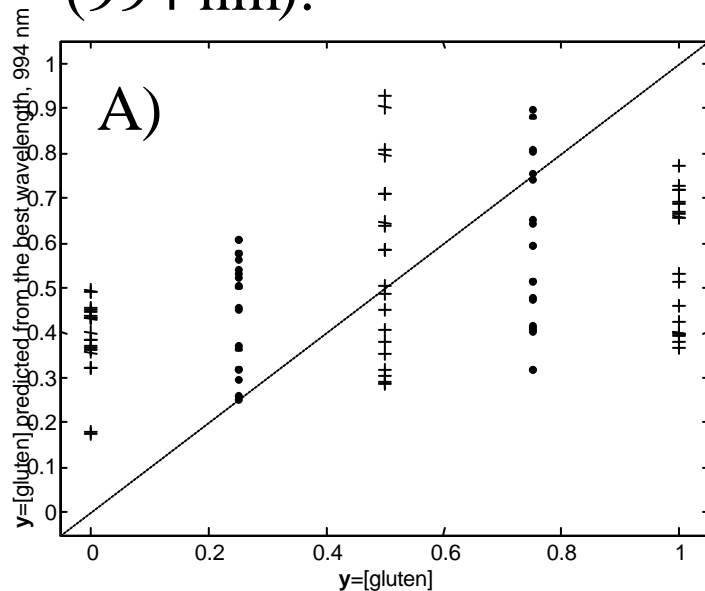
Light Scattering and Light Absorbance Separated by Extended Multiplicative Signal Correction.

Application to Near-Infrared Transmission Analysis of Powder Mixtures . Anal. Chem. 2003; **75** (3) pp 394 – 404.

# Calibration for $y$ =gluten fraction for $\mathbf{X}$ NIR $\log(1/T)$

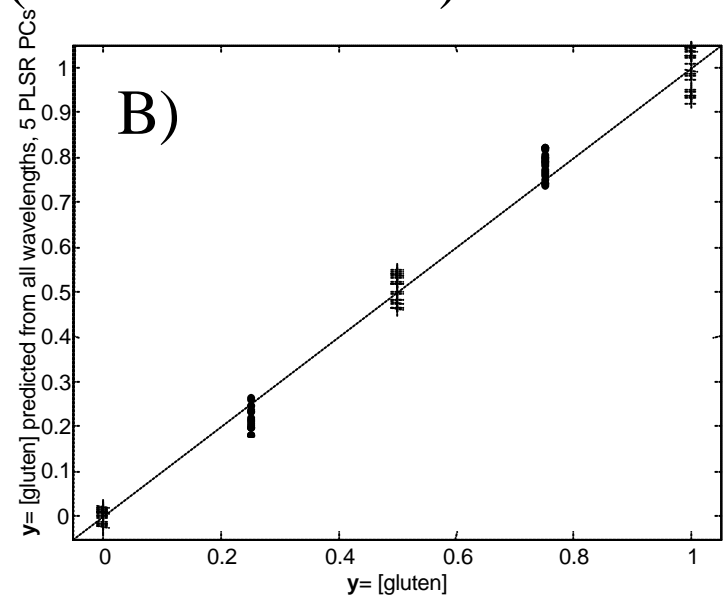
Best univariate calibration

(994 nm):

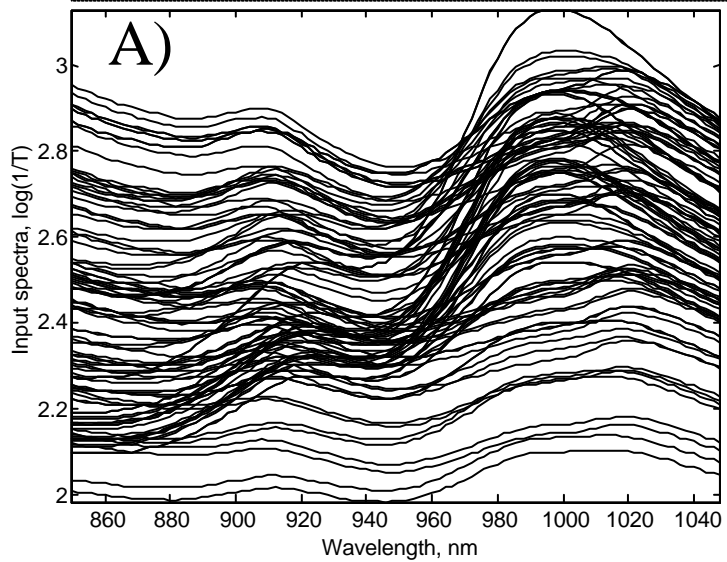


Best multivariate calibration

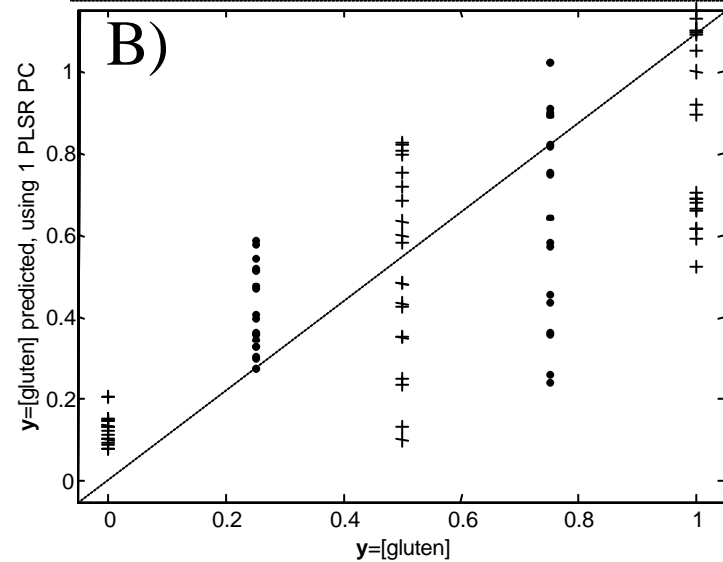
(via 5 PLSR PCs):



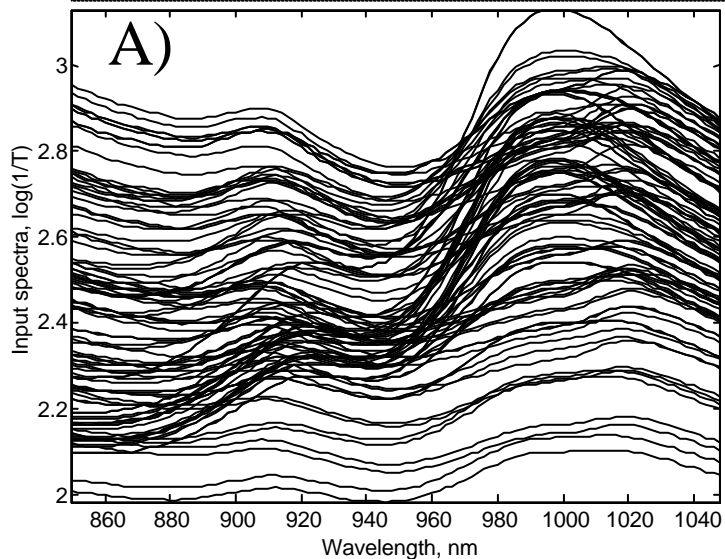
No Pre-processing;



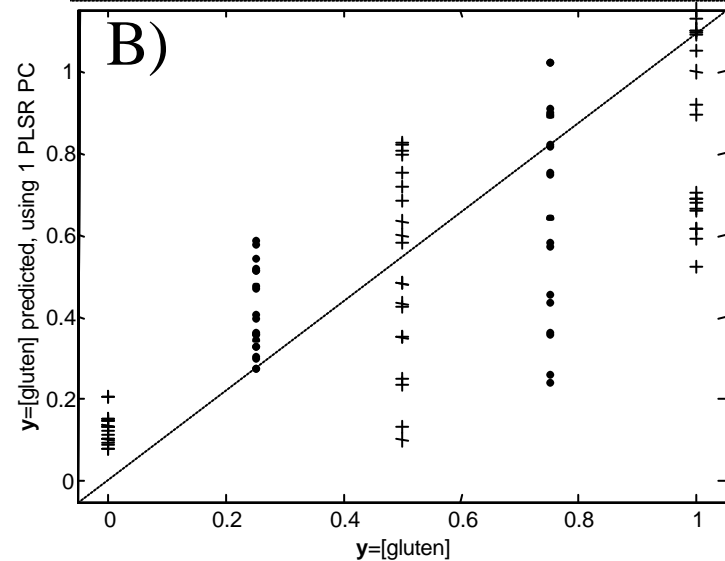
Multivariate calibration (via 1 PLSR PC)



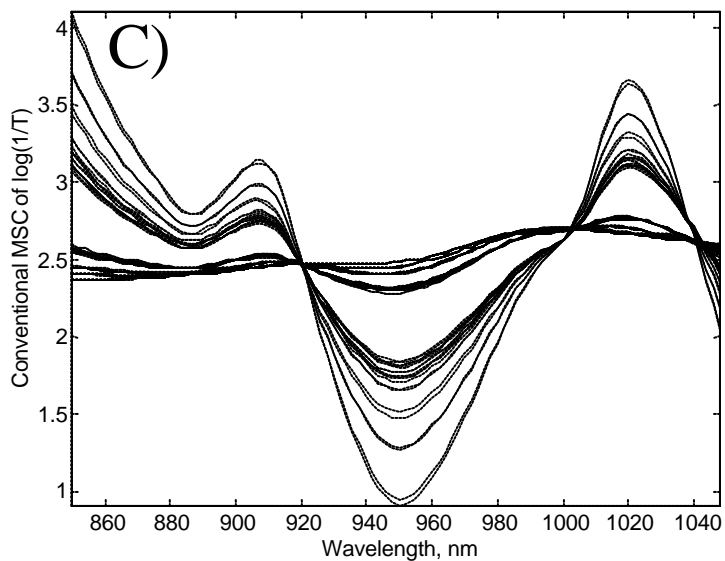
No Pre-processing:



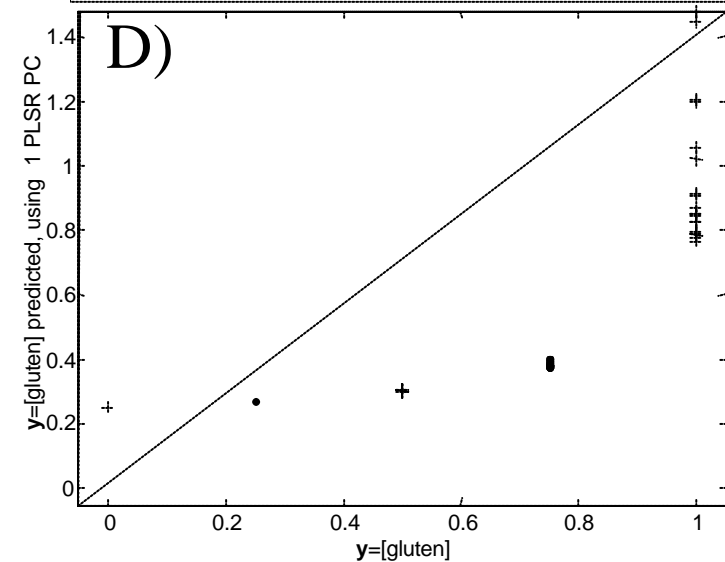
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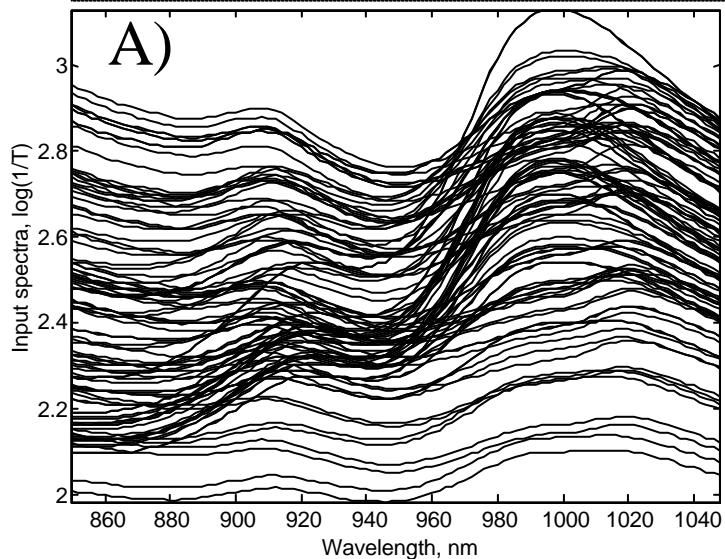
Traditional MSC pre-processing:



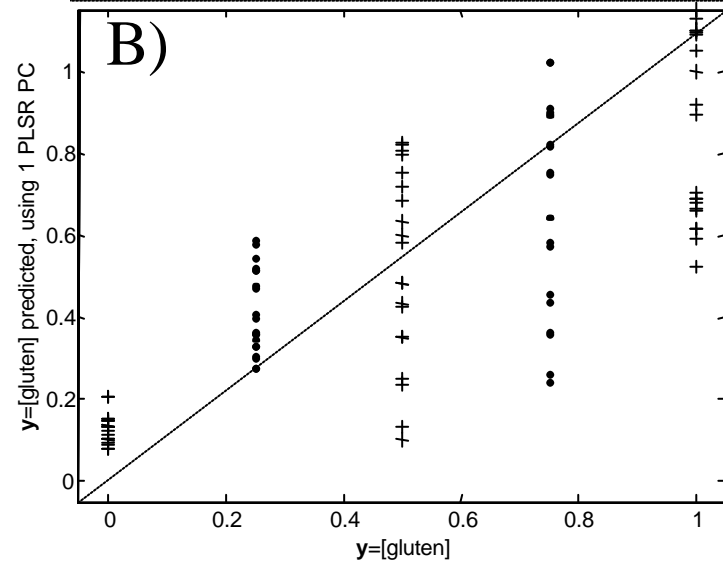
Multivariate calibration (via 1 PLSR PC):



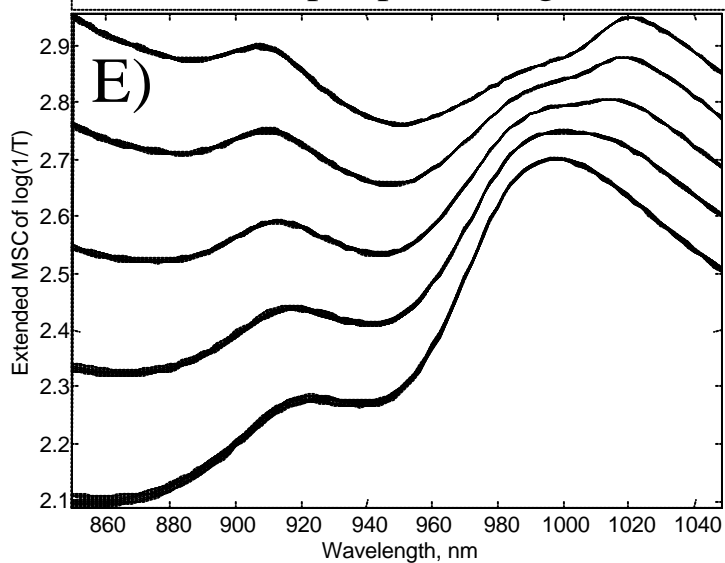
No Pre-processing;



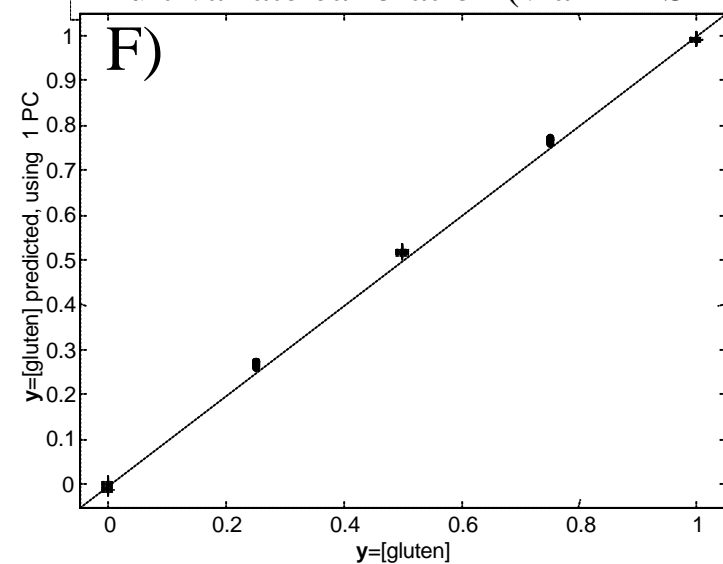
Multivariate calibration (via 1 PLSR PC)



New EMSC pre-processing:



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## EMSC theory:

Each input spectrum  $\mathbf{z}_i$  is modelled and corrected:



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- \* Estimate parameters in model  $f()$ .

- \* Correct spectra by EMSC:

$$\mathbf{z}_{i,\text{chem}} \approx \mathbf{z}_{i,\text{corrected}} = f^{-1}(\mathbf{z}_i, \text{component spectra, scattering spectra})$$

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Improve the model, the correction, the parameter estimation, the component spectra



Ideal chemical model: Additive model (Beer's law):

$$\begin{aligned}\mathbf{z}_{i,\text{chem}} &= c_{i1}\mathbf{k}_1' + \dots + c_{ij}\mathbf{k}_j' + \dots + c_{iJ}\mathbf{k}_J' \\ &= \mathbf{c}_i\mathbf{K}'\end{aligned}$$

where  $c_{ij}$  = conc.,  $\mathbf{k}_j$  = absorptivity of constituent  $j$

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Expressed as deviations around a reference spectrum:

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The EMSC model of physical interferants:

$$\mathbf{z}_i \approx a_i\mathbf{1} + b_i\mathbf{z}_{i,\text{chem}} + d_i\mathbf{l} + e_i\mathbf{l}^2$$

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Linear model for estimation:

$$\mathbf{z}_i = a_i\mathbf{1} + \underbrace{b_i\mathbf{m}' + b_i\mathbf{c}_i\mathbf{K}'}_{\text{arrow from } b_i\mathbf{z}_{i,\text{chem}} \text{ in previous equation}} + d_i\mathbf{l} + e_i\mathbf{l}^2 + \mathbf{e}_i$$

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The EMSC correction:

$$\mathbf{z}_{i,\text{corrected}} = (\mathbf{z}_i - a_i\mathbf{1} - d_i\mathbf{l} - e_i\mathbf{l}^2)/b_i$$

Deviations around a reference spectrum  $\mathbf{m}$   
(e.g. the mean spectrum):

$$\mathbf{z}_{i,\text{chem}} = \mathbf{m}' + c_{i1}\mathbf{k}_1' + c_{i2}\mathbf{k}_2' + \dots + c_{iJ}\mathbf{k}_J'$$

where concentrations  $c_{ij}$  represent deviations around the unknown concentrations in the reference

**Only two constituents**

$$\mathbf{z}_{i,\text{chem}} = c_{i1}\mathbf{k}_1' + c_{i2}\mathbf{k}_2'$$



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$$\mathbf{z}_{i,\text{chem}} = c_{i1}\mathbf{k}_1' + c_{i2}\mathbf{k}_2' = \mathbf{m}' + c_i\mathbf{K}'$$

where  $\mathbf{K} = \mathbf{k}_1 - \mathbf{k}_2$  difference spectrum

# Estimation of model parameters by weighted least squares:

$$\mathbf{z}_i = a_i \mathbf{1}' + b_i \mathbf{m}' + \mathbf{h}_i \mathbf{K}' + d_i \mathbf{l} + e_i \mathbf{l}^2 + \mathbf{e}_i$$

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$$\mathbf{M} = [\mathbf{1}'; \mathbf{m}'; \mathbf{K}'; \mathbf{l}; \mathbf{l}^2]$$

$$\mathbf{p}_i = [a_i, b_i, \mathbf{h}_i, d_i, e_i]$$

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$$\mathbf{p}_i = \mathbf{z}_i \mathbf{V} \mathbf{M}' (\mathbf{M} \mathbf{V} \mathbf{M}')^{-1}$$

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$$\mathbf{z}_i = \mathbf{p}_i \mathbf{M} + e_i \quad \mathbf{V} = \text{weight matrix}$$

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$$\mathbf{p}_i = \mathbf{z}_i \mathbf{V} \mathbf{M}' (\mathbf{M} \mathbf{V} \mathbf{M}')^{-1}$$

$$e_i = \mathbf{z}_i - \mathbf{p}_i \mathbf{M}$$

**EMSC:**

$$\mathbf{z}_{i,\text{chem}} = \mathbf{m}' + c_{i1}\mathbf{k}_1' + c_{i2}\mathbf{k}_2' + \dots + c_{iJ}\mathbf{k}_J'$$

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**MSC:** *Ignore*  $c_{i1}\mathbf{k}_1' + c_{i2}\mathbf{k}_2' + \dots + c_{iJ}\mathbf{k}_J'$

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*Further simplification:*  $\mathbf{d}_i \approx \mathbf{0}$

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**EMSC:**

$$\mathbf{z}_{i,\text{chem}} = \mathbf{m}' + c_{i1}\mathbf{k}_1' + c_{i2}\mathbf{k}_2' + \dots + c_{iJ}\mathbf{k}_J'$$

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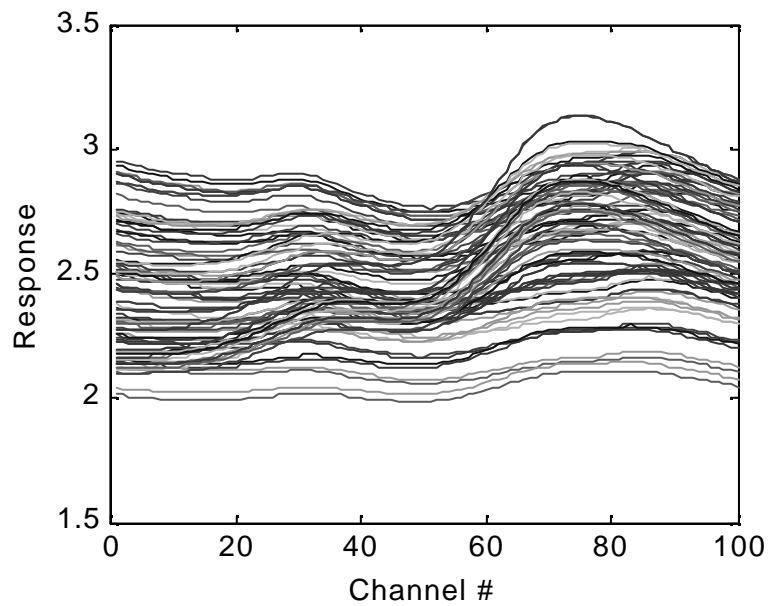
$$\mathbf{z}_{i,\text{chem}} \approx \mathbf{m}'$$

**MSC model:**

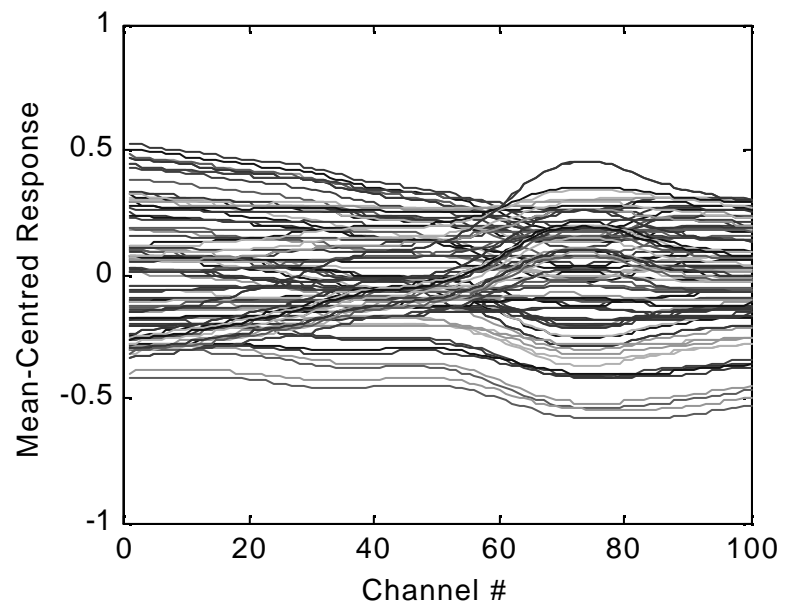
$$\mathbf{z}_i = a_i\mathbf{1}' + b_i\mathbf{m}' + \mathbf{e}_i$$

$$\mathbf{z}_{i,\text{corrected}} = (\mathbf{z}_i - a_i\mathbf{1}')/b_i$$

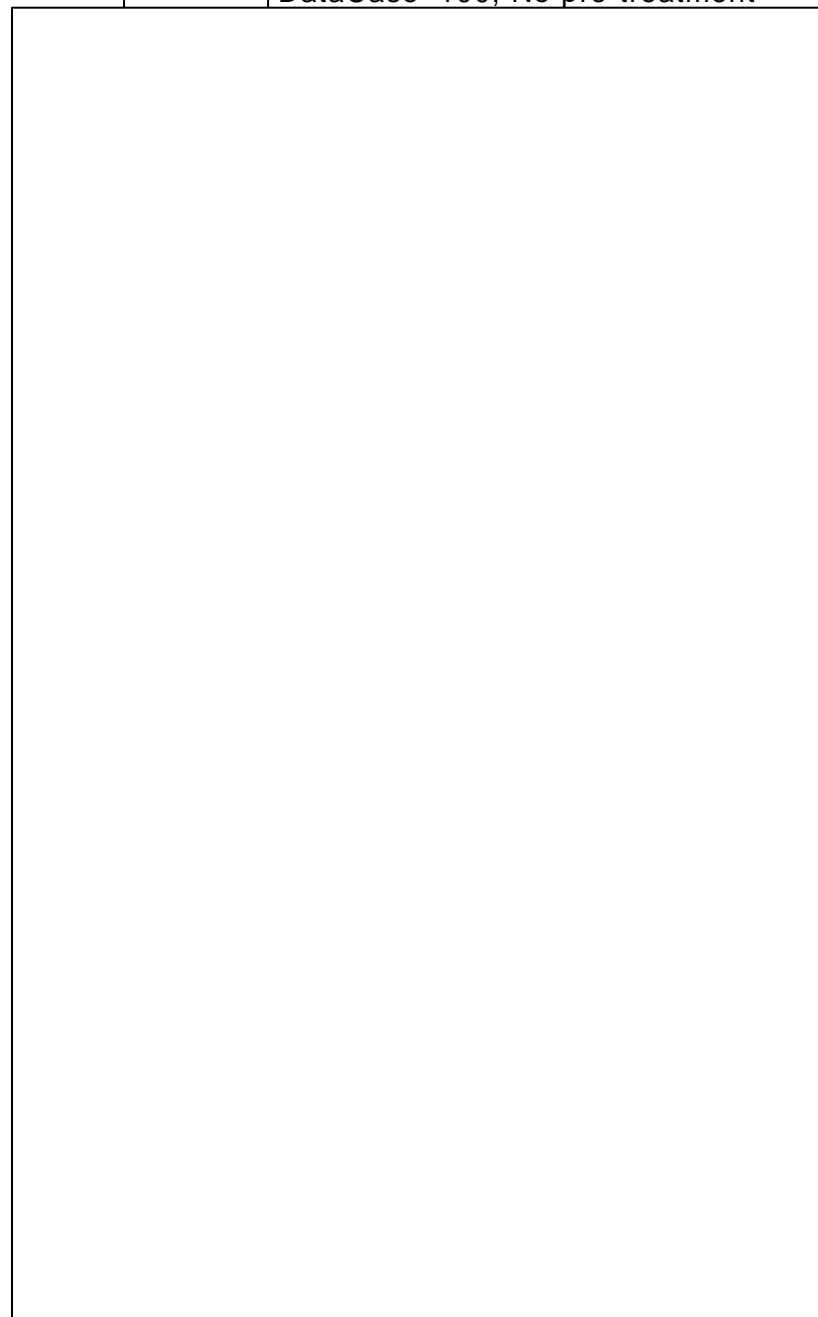
Input, EMSC<sub>z</sub>.MAT



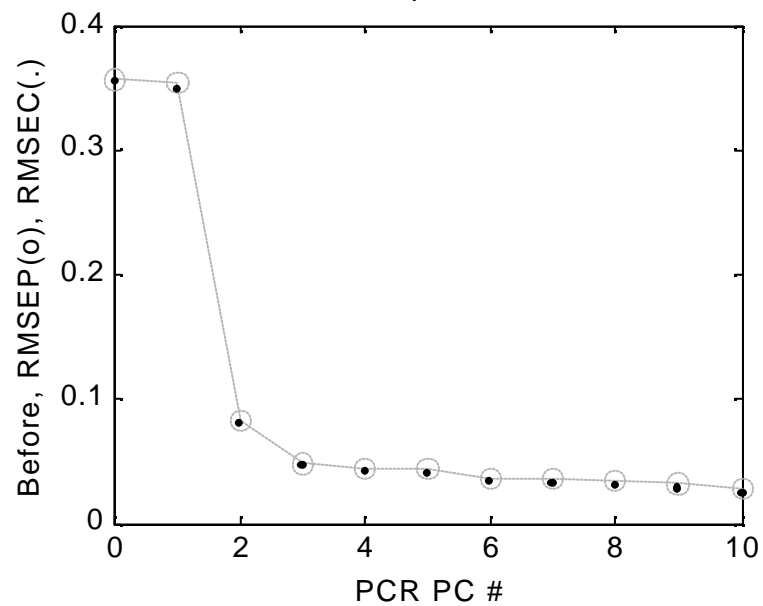
Input, EMSC<sub>z</sub>.MAT



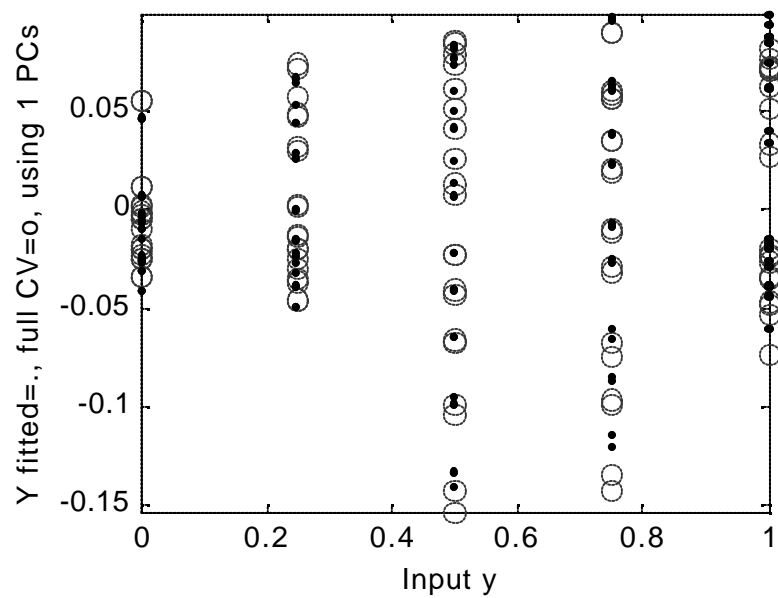
DataCase=100, No pre-treatment



DataCase=100 No pre-treatment, before



Cal. for y from input Z,  $r_{CV}=0.046$



MSC: Conventional modelling of  
offset  $a_i$  (pathlength or scaling) and  
slope  $b_i$  (baseline):

Model:

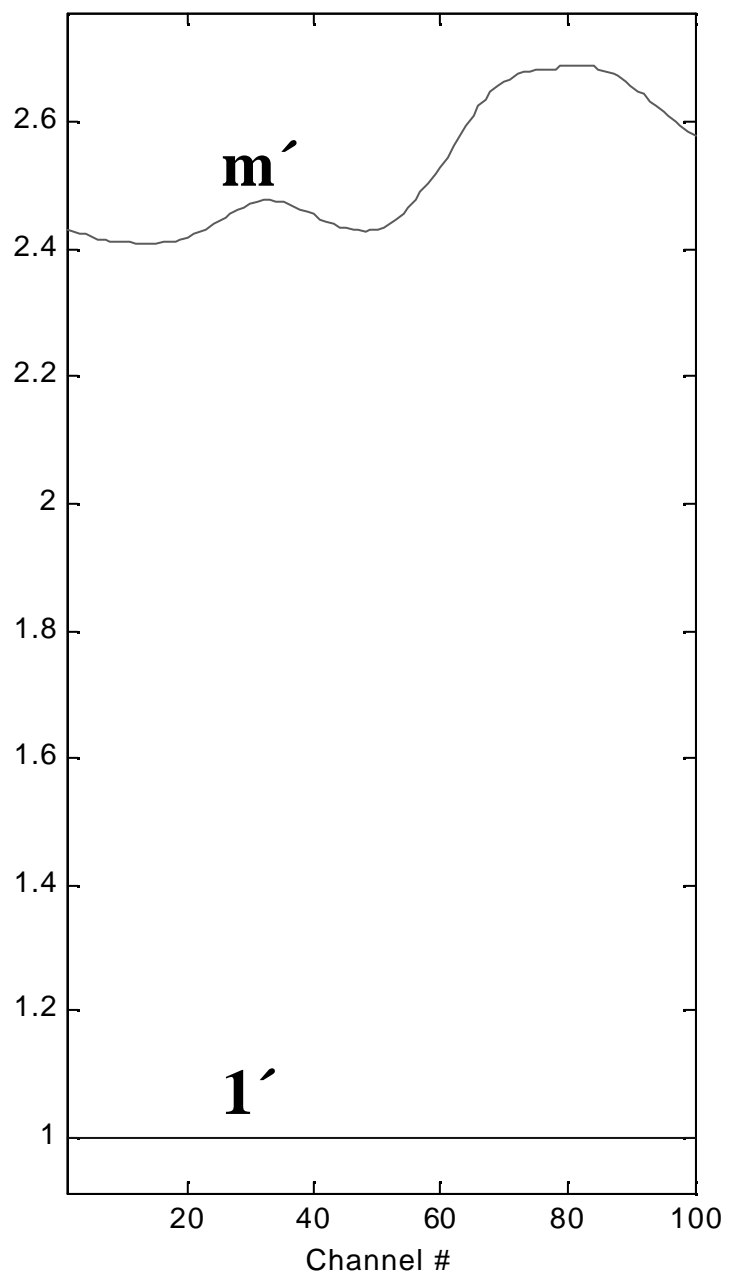
$$\mathbf{z}_i = a_i \mathbf{1} + b_i \mathbf{m} + \mathbf{e}_i$$

Estimate  $a_i$  and  $b_i$  from  $\mathbf{z}_i$ ,  $\mathbf{1}$  and  $\mathbf{m}$

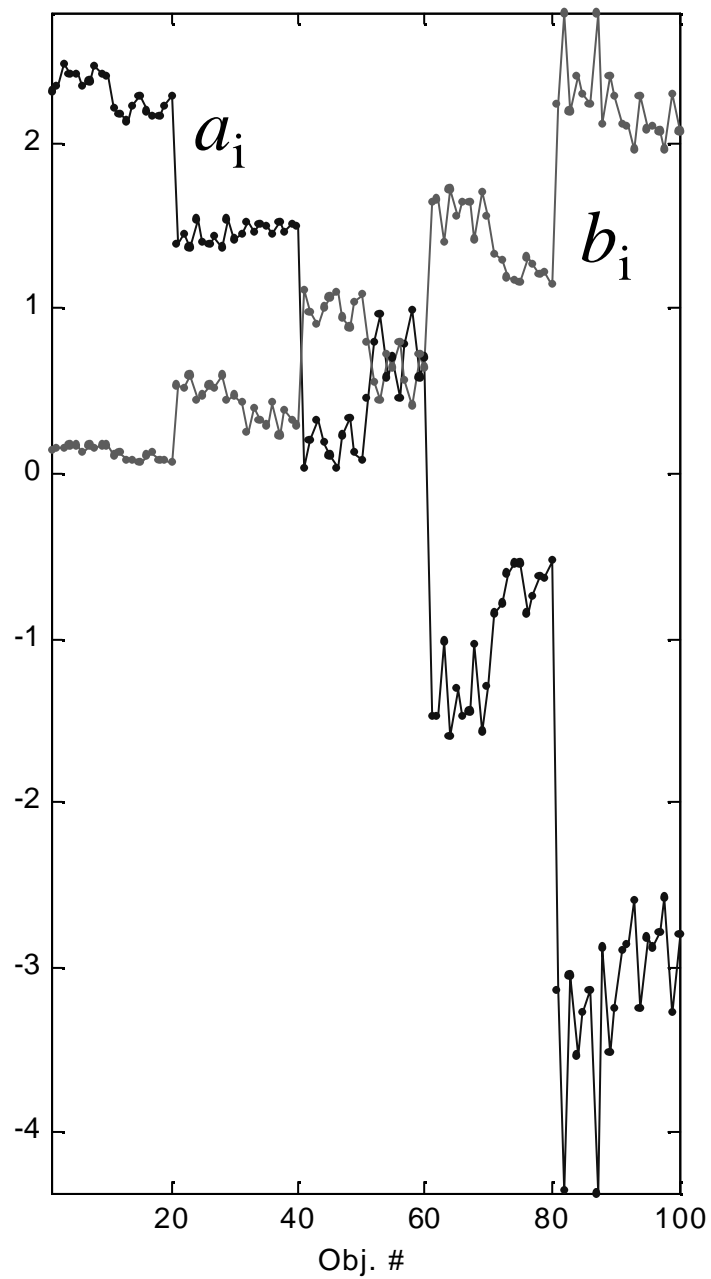
MSC correction:

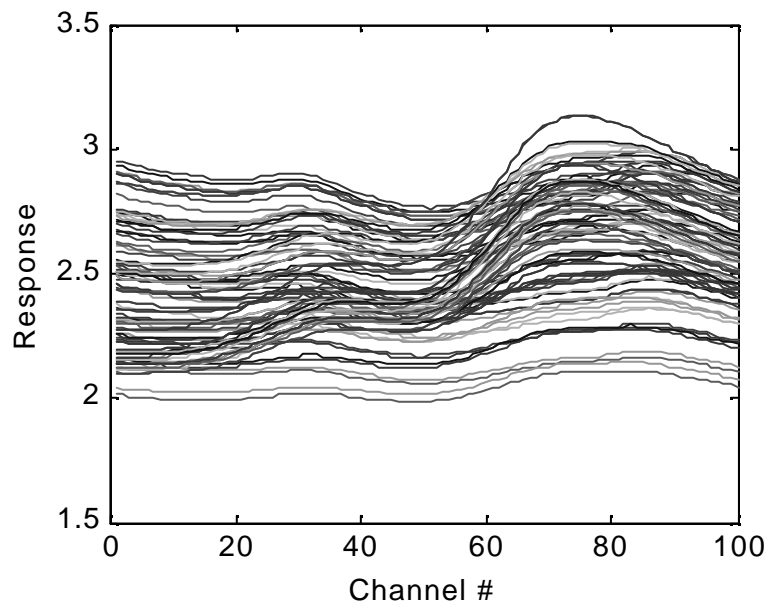
$$\mathbf{z}_{i,\text{corrected}} = (\mathbf{z}_i - a_i)/b_i$$

Model spectra

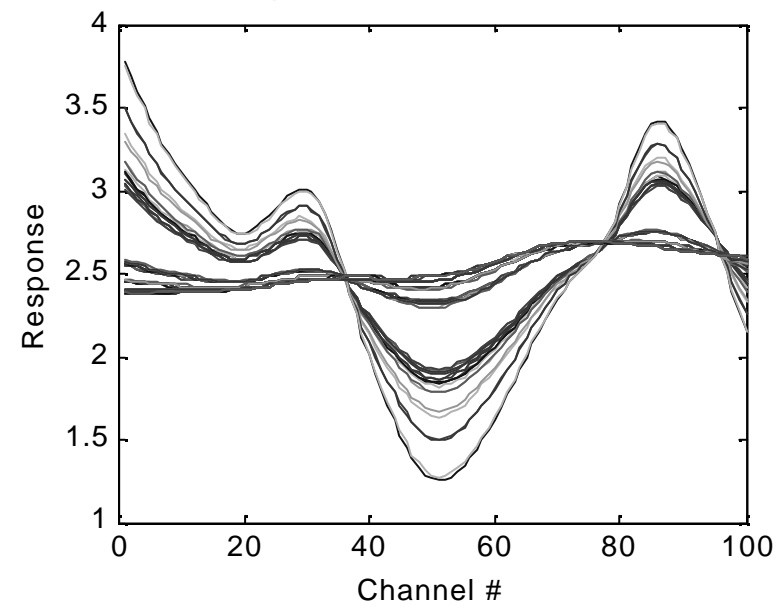


All parameter estimates together



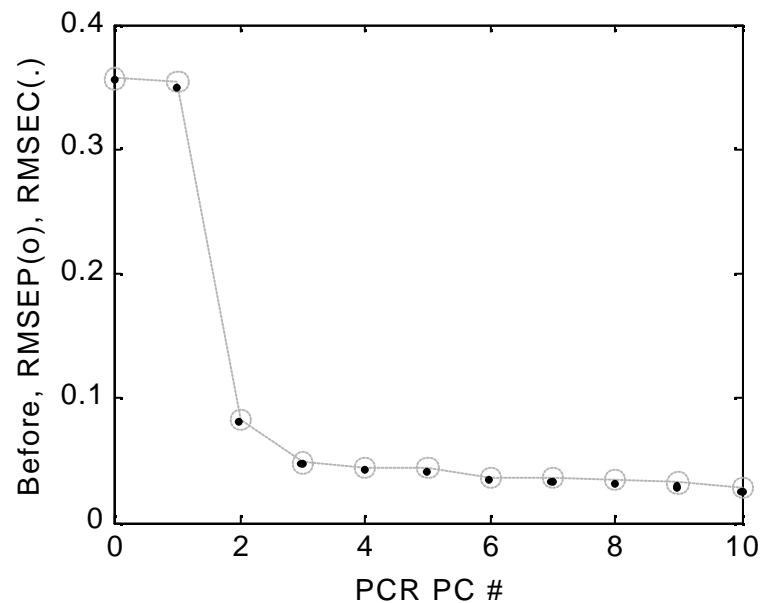
Input, EMSC<sub>z</sub>.MAT

Output, DataCase=102, MSC

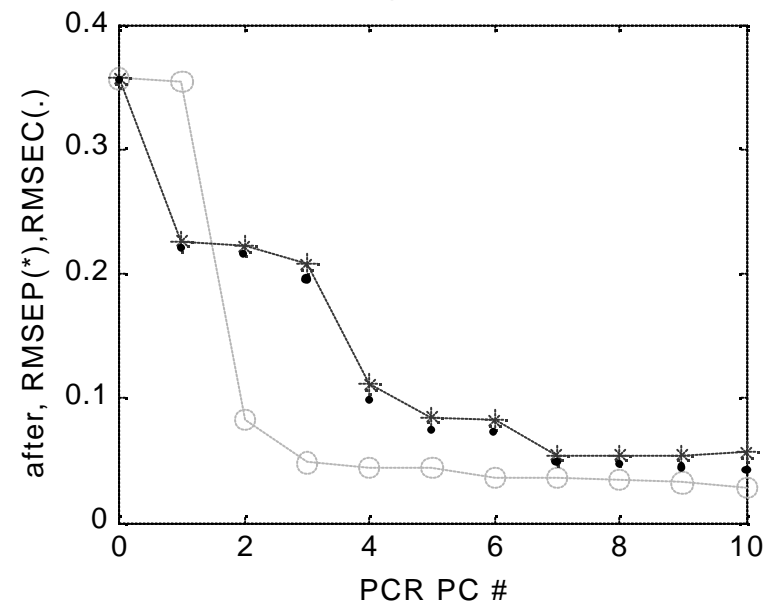
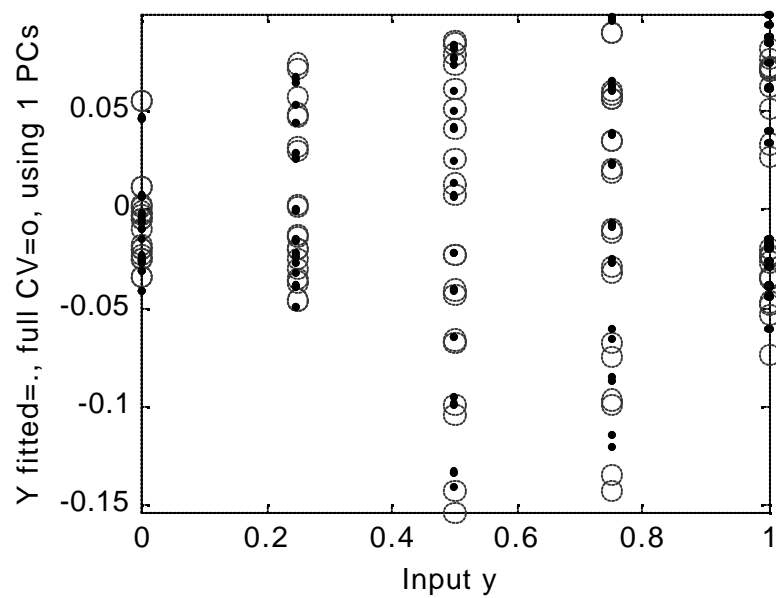
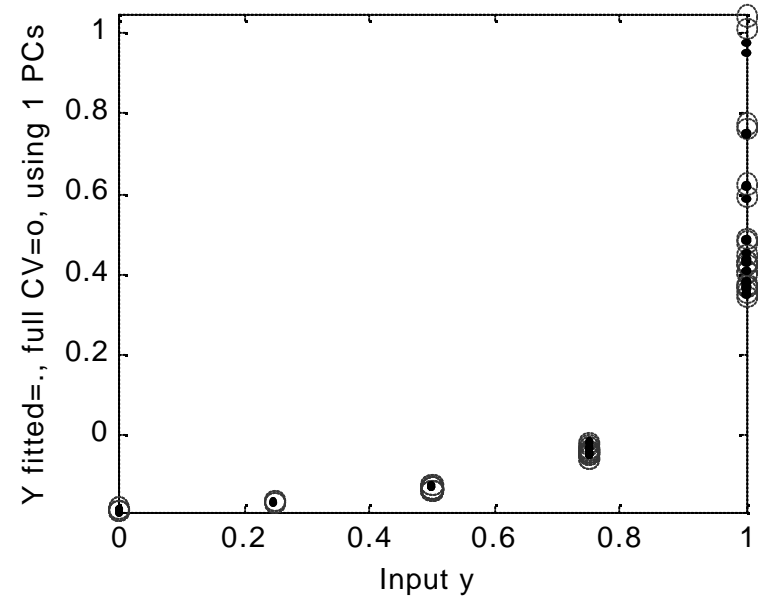


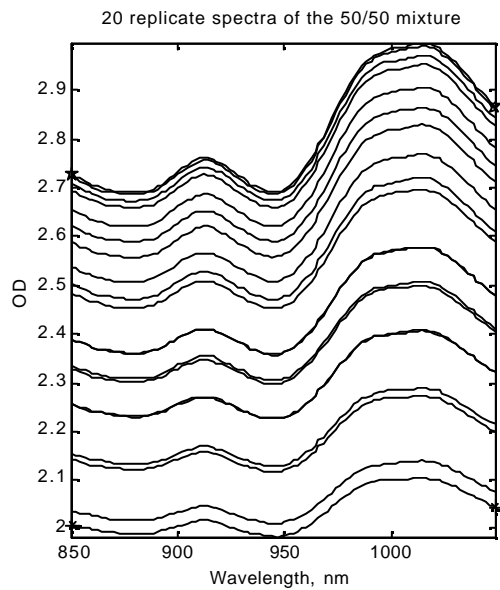


DataCase=102 MSC, before



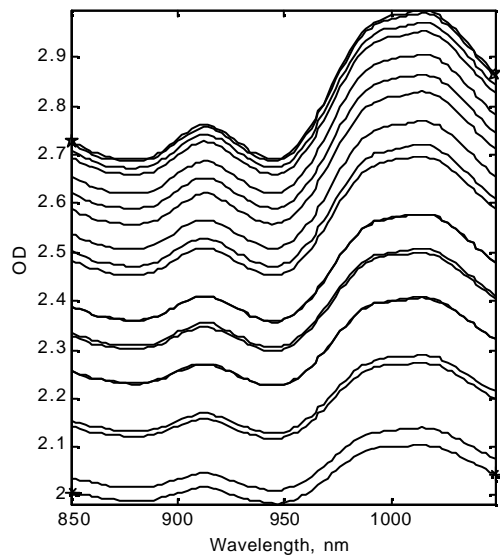
after pre-treatment

Cal. for y from input Z,  $r_{CV}=0.046$ Cal. for y after EMSC/EISC,  $r_{CV}=0.772$ 

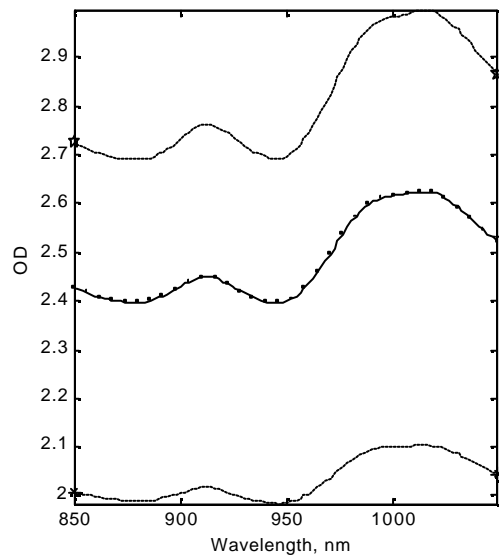


20 samples of identical chemical composition (50/50 gluten/starch), but different path length, powder packing etc)

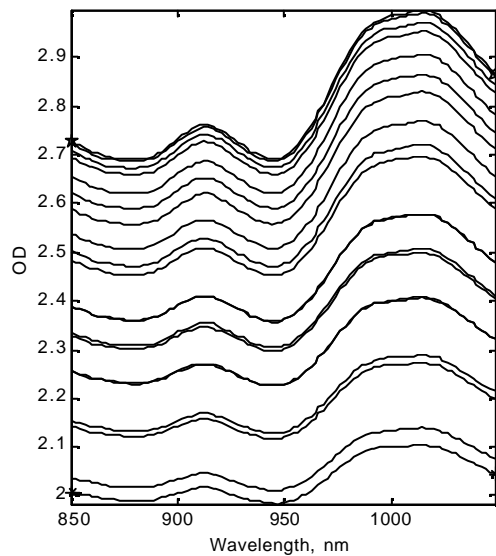
20 replicate spectra of the 50/50 mixture



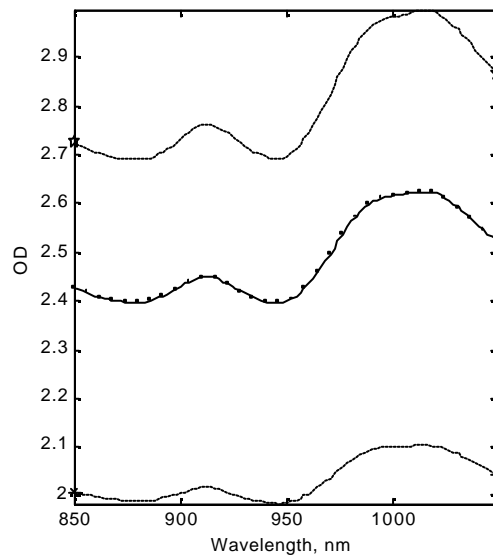
Mean (m) and spectra  $z_{41}$  and  $z_{58}$



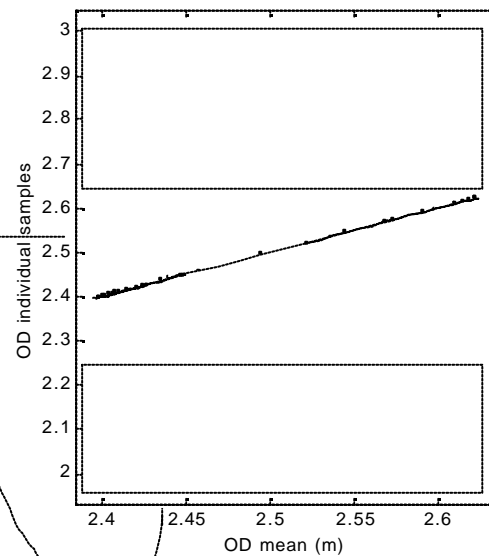
20 replicate spectra of the 50/50 mixture



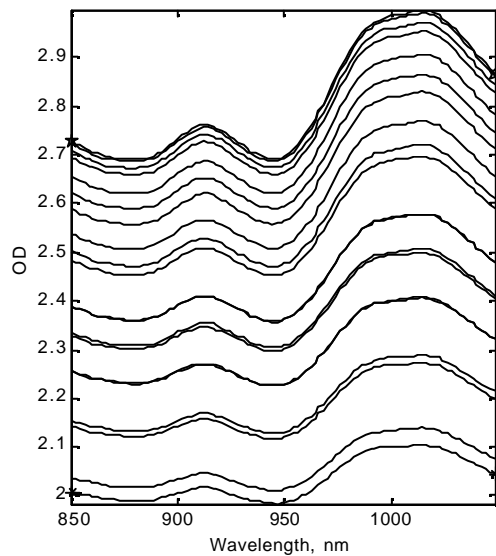
Mean ( $m$ ) and spectra  $z_{41}$  and  $z_{58}$



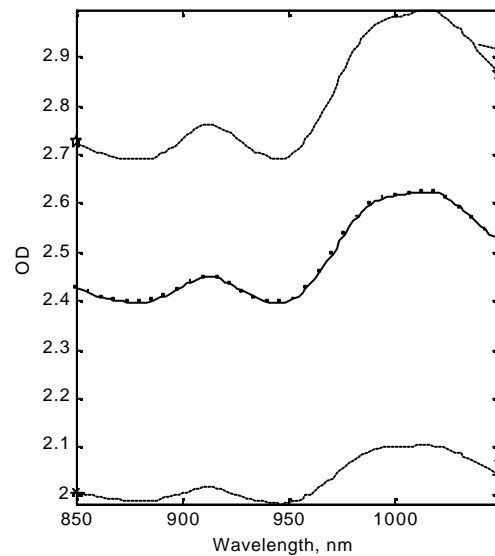
Mean ( $m$ ) and spectra  $z_{41}$  and  $z_{58}$  plotted against  $m$



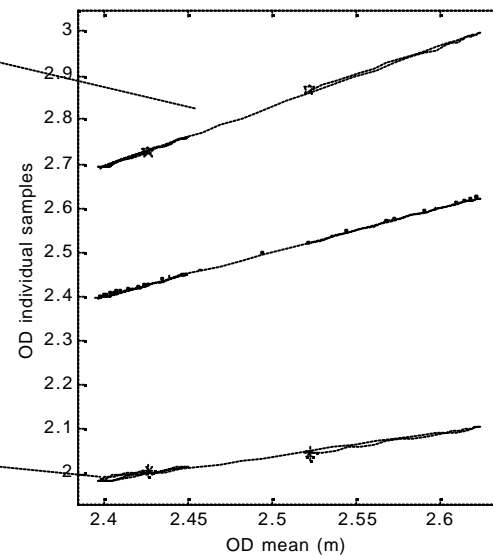
20 replicate spectra of the 50/50 mixture



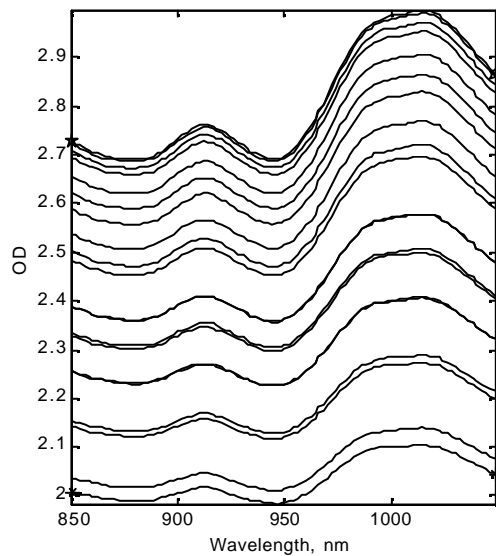
Mean (m) and spectra  $z_{41}$  and  $z_{58}$



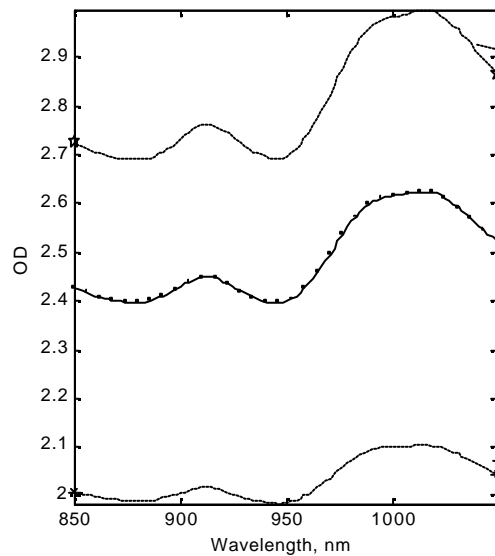
Mean (m) and spectra  $z_{41}$  and  $z_{58}$  plotted against m



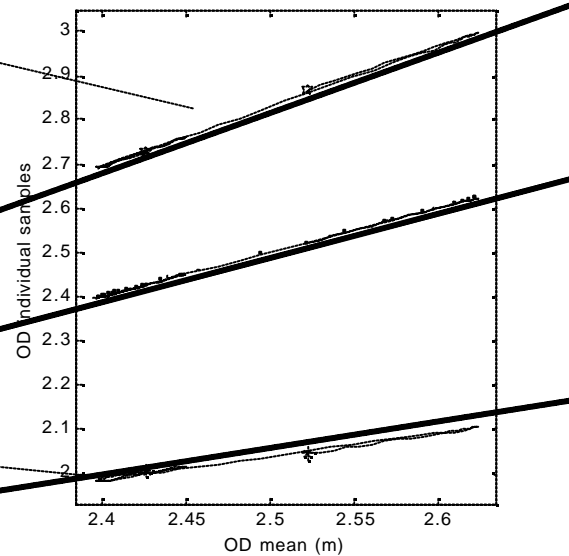
20 replicate spectra of the 50/50 mixture



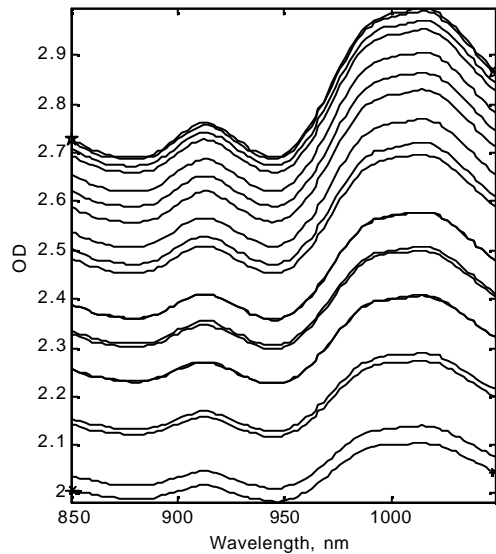
Mean (m) and spectra  $z_{41}$  and  $z_{58}$



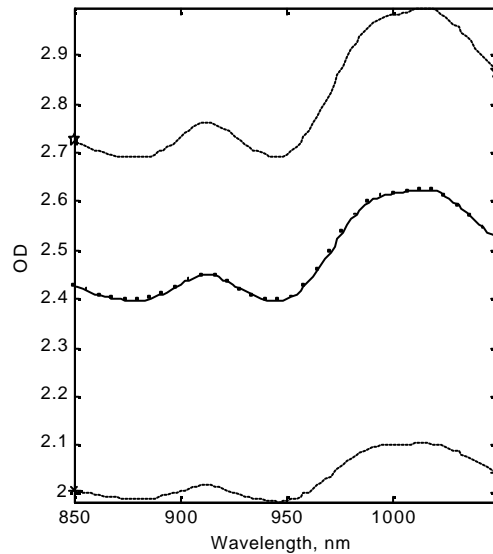
Mean (m) and spectra  $z_{41}$  and  $z_{58}$  plotted against m



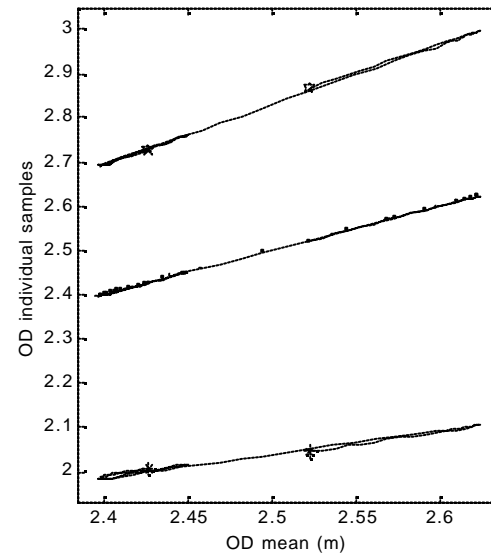
20 replicate spectra of the 50/50 mixture



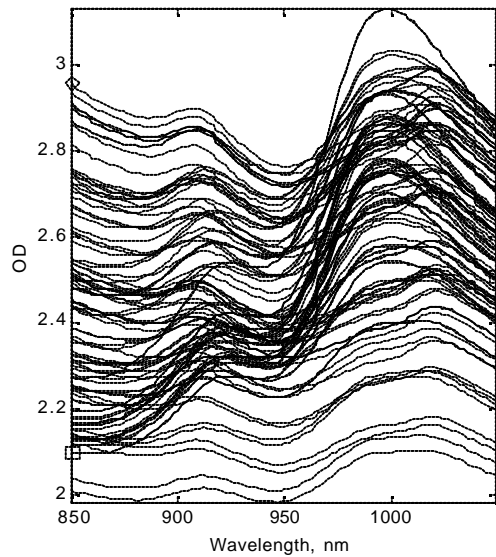
Mean ( $m$ ) and spectra  $z_{41}$  and  $z_{58}$



Mean ( $m$ ) and spectra  $z_{41}$  and  $z_{58}$  plotted against  $m$

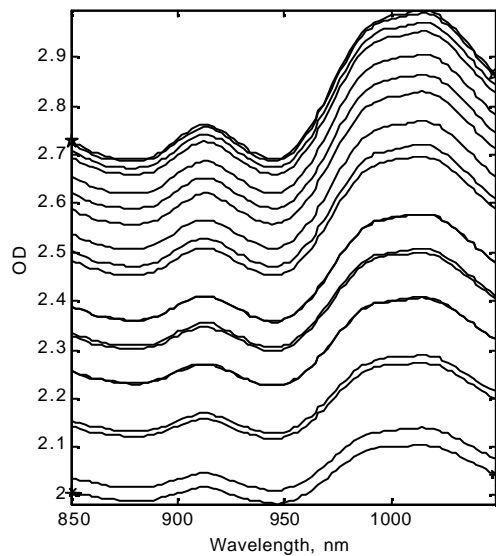


20 replicate spectra for each of the 5 mixtures

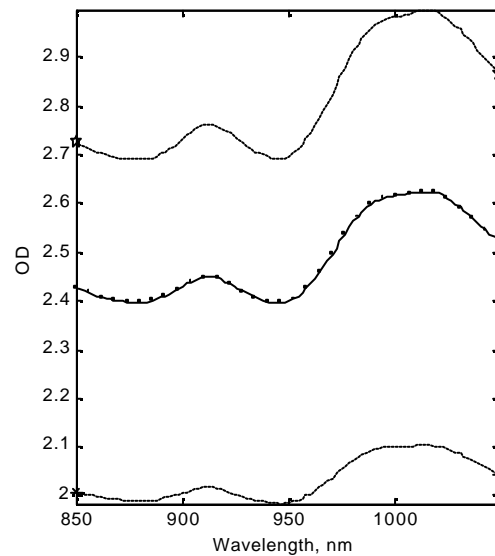


All 5 chemical mixtures

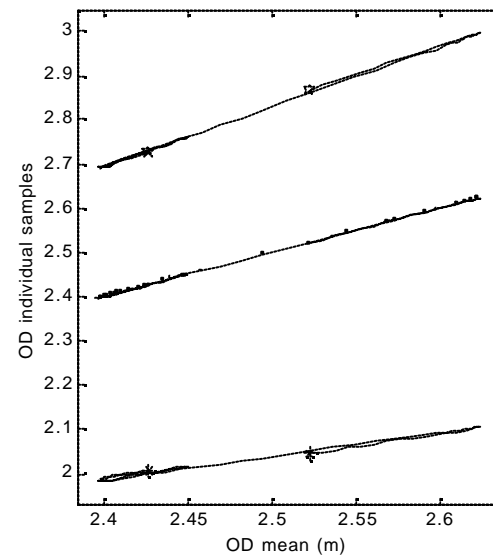
20 replicate spectra of the 50/50 mixture



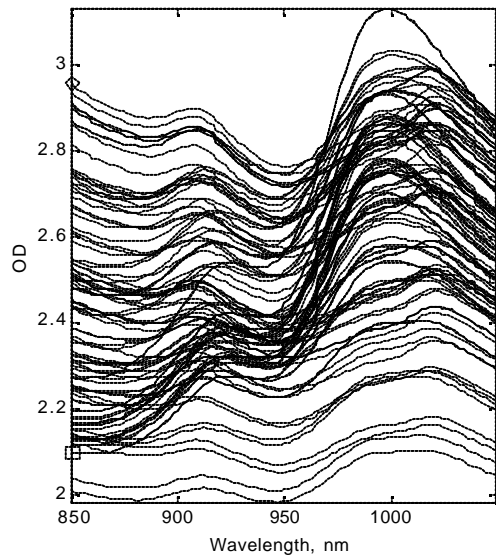
Mean ( $m$ ) and spectra  $z_{41}$  and  $z_{58}$



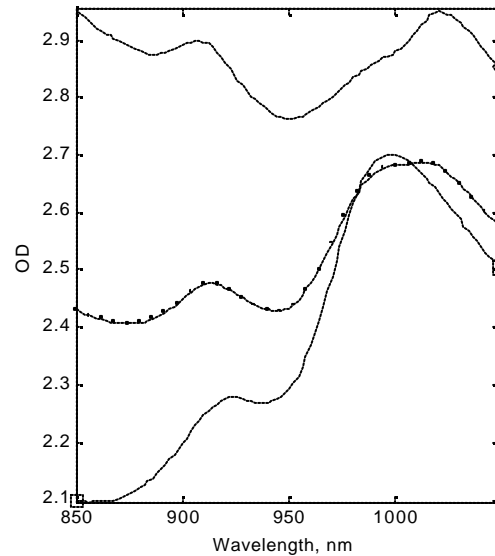
Mean ( $m$ ) and spectra  $z_{41}$  and  $z_{58}$  plotted against  $m$



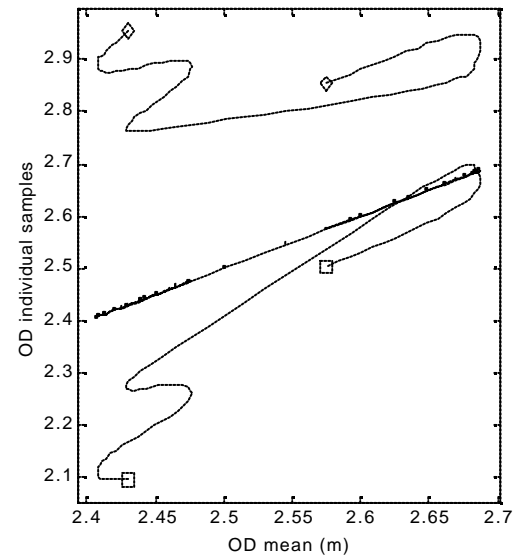
20 replicate spectra for each of the 5 mixtures



Mean ( $m$ ) and spectra  $z_3$  and  $z_{93}$

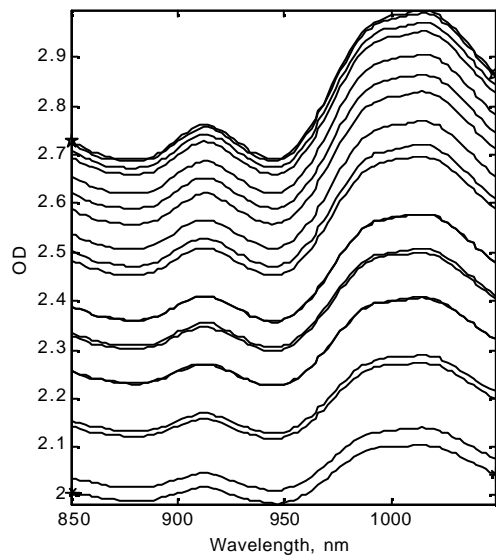


Mean ( $m$ ) and spectra  $z_3$  and  $z_{93}$  plotted against  $m$

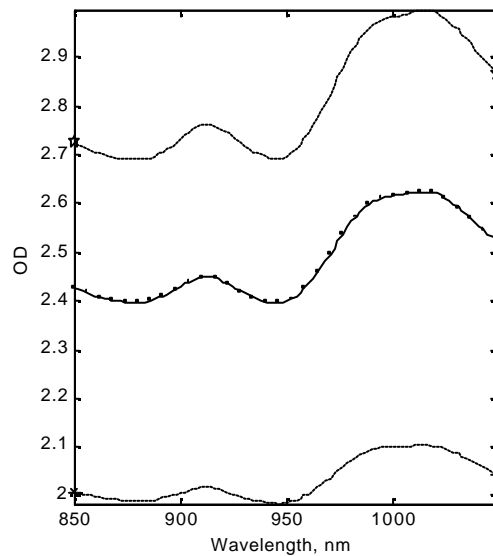




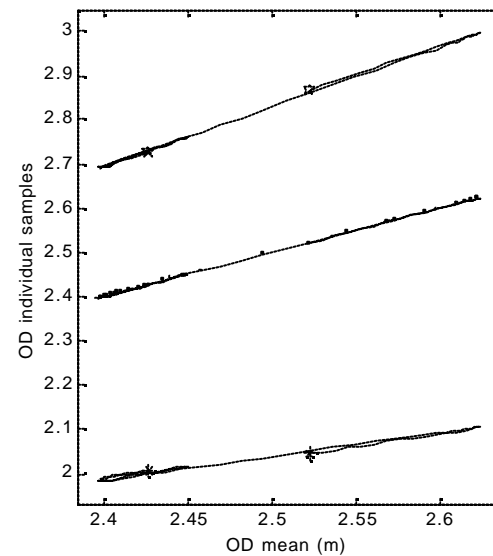
20 replicate spectra of the 50/50 mixture



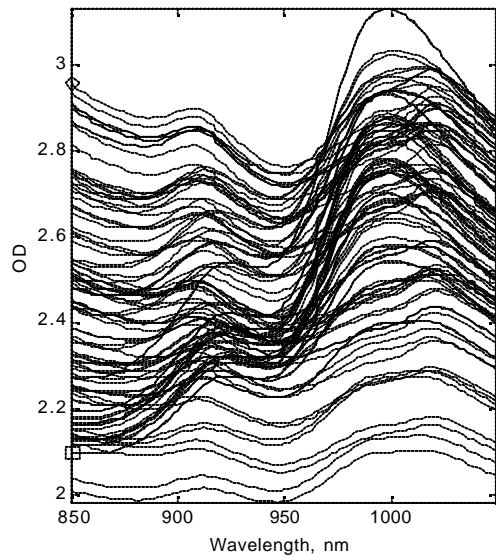
Mean ( $m$ ) and spectra  $z_{41}$  and  $z_{58}$



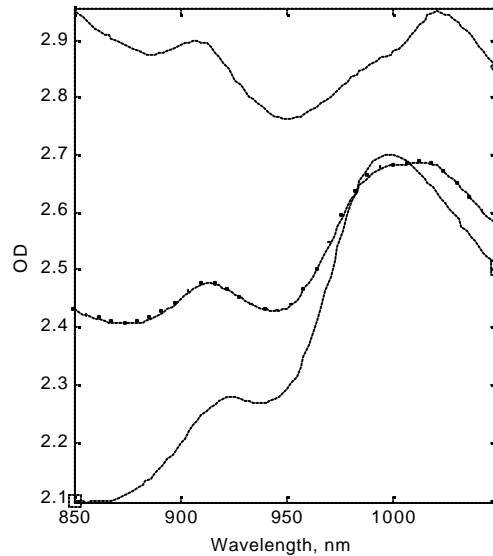
Mean ( $m$ ) and spectra  $z_{41}$  and  $z_{58}$  plotted against  $m$



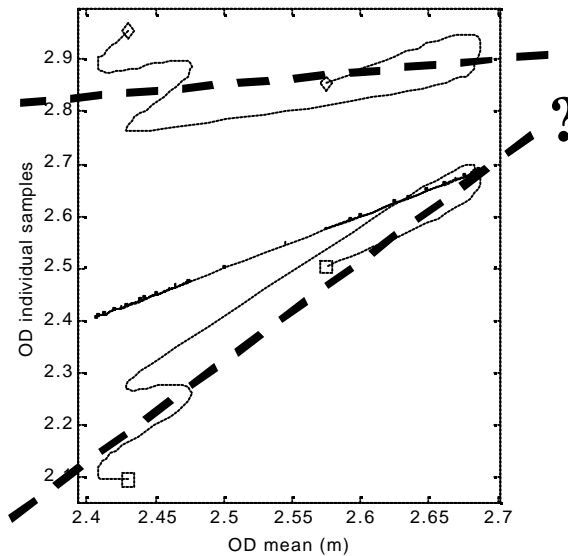
20 replicate spectra for each of the 5 mixtures



Mean ( $m$ ) and spectra  $z_3$  and  $z_{93}$



Mean ( $m$ ) and spectra  $z_3$  and  $z_{93}$  plotted against  $m$



**EMSC Default:** Physical model,  
but no chemical model parameters

**Ideal chemical model: Like MSC:**

$$\mathbf{z}_{i,\text{chem}} = \mathbf{m}' + d_i'$$

**The EMSC model of physical interferants:**

$$\mathbf{z}_i \approx a_i \mathbf{1}' + b_i \mathbf{z}_{i,\text{chem}} + d_i \mathbf{1} + e_i \mathbf{1}^2$$

$$\text{EMSC: } \mathbf{z}_i = a_i \mathbf{1}' + b_i \mathbf{m}' + h_i \mathbf{k}' + d_i \mathbf{1} + e_i \mathbf{1}^2 + \mathbf{e}_i$$

$$\text{EMSC correction: } \mathbf{z}_{i,\text{corrected}} = (\mathbf{z}_i - a_i - d_i \mathbf{1} - e_i \mathbf{1}^2) / b_i$$

# EMSC Toolbox for Matlab:

**<http://www.models.kvl.dk/source/EMSCtoolbox/index.asp>**

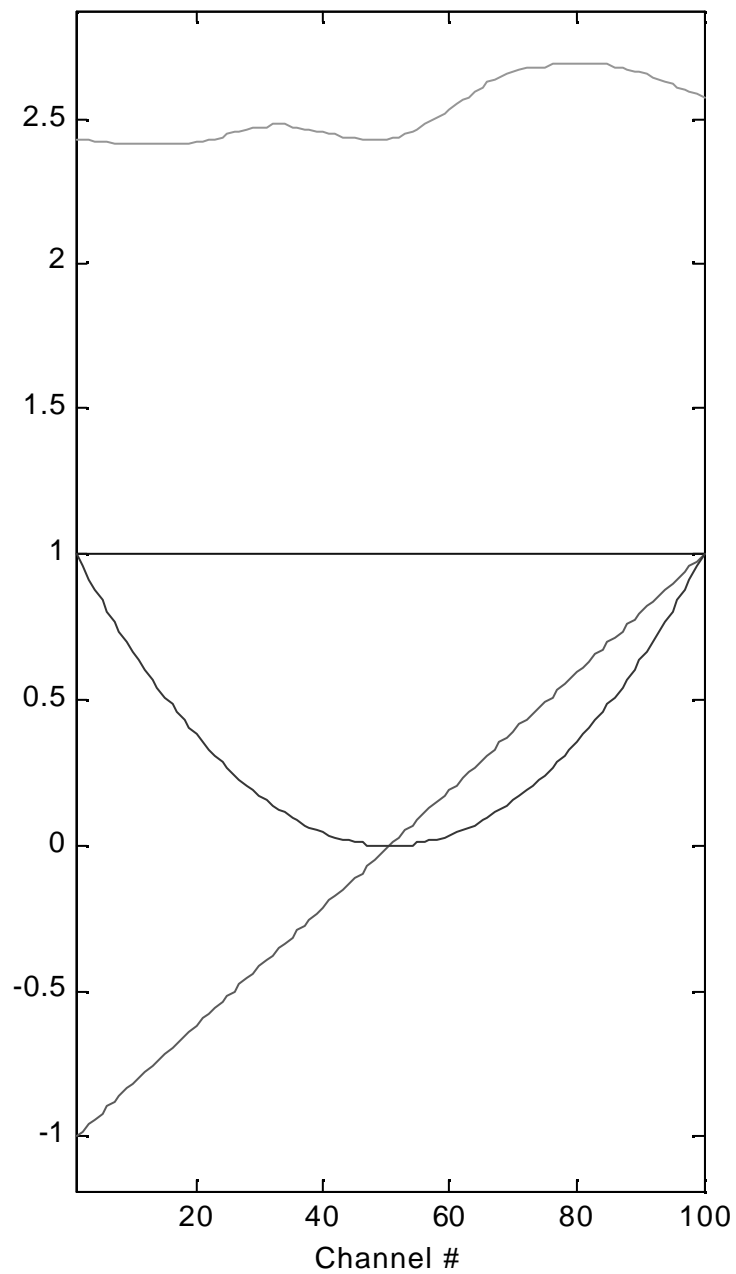
Pre-defined default EMSC in the software:

```
...  
elseif DataCase==(103)  
    DataCaseName='EMSC physical,default'  
elseif ...
```

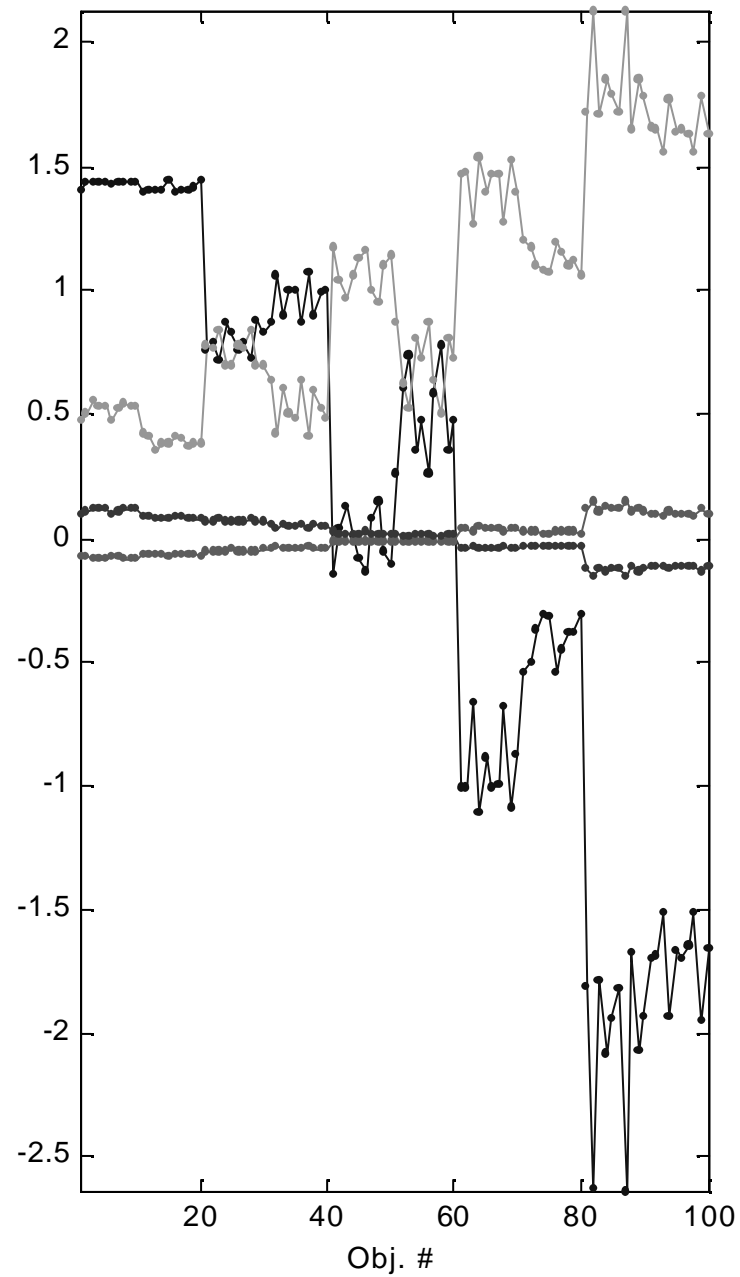
Running the program:

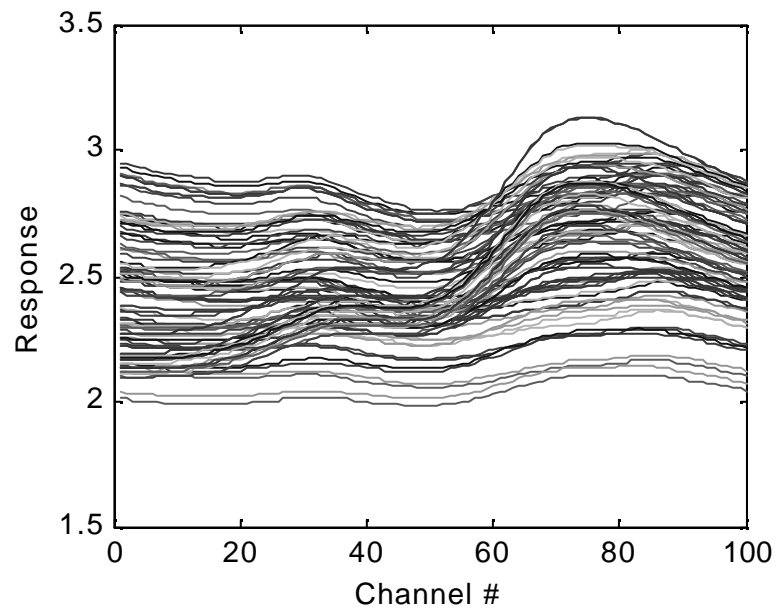
DataCase=? 103

Model spectra

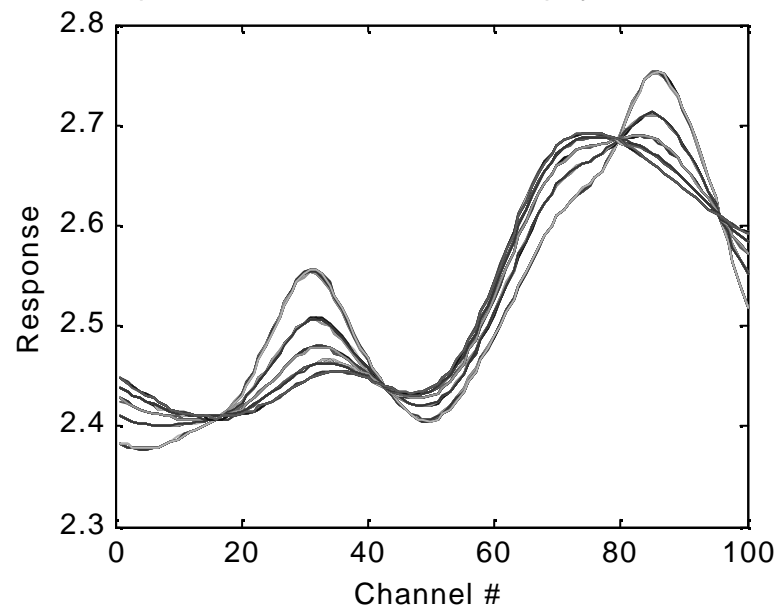
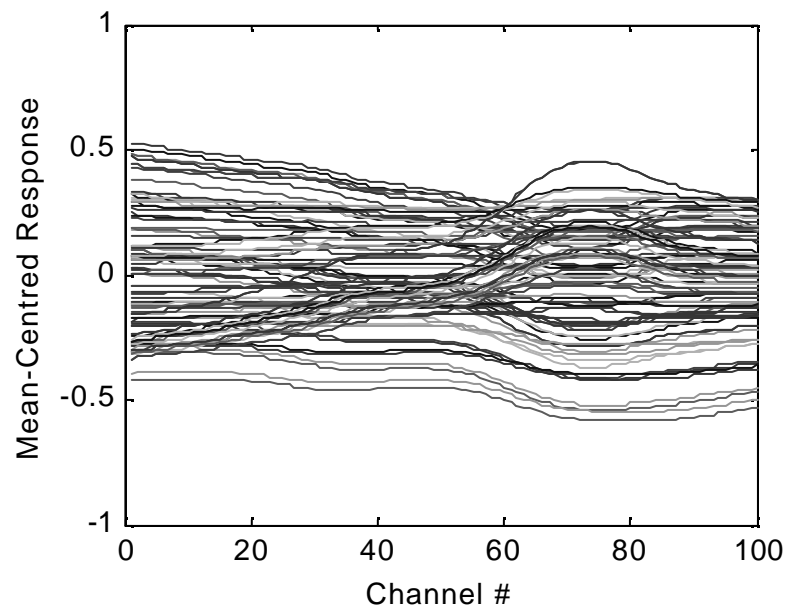


All parameter estimates together

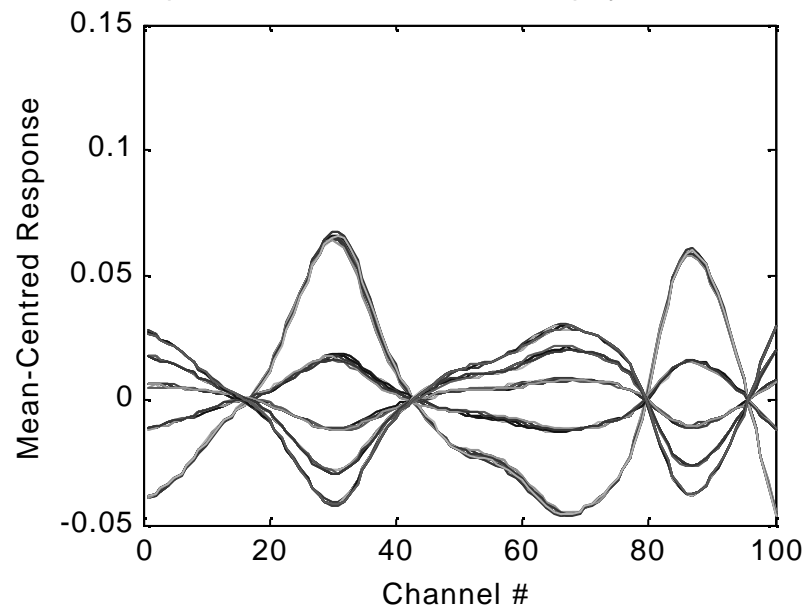


Input, EMSC<sub>z</sub>.MAT

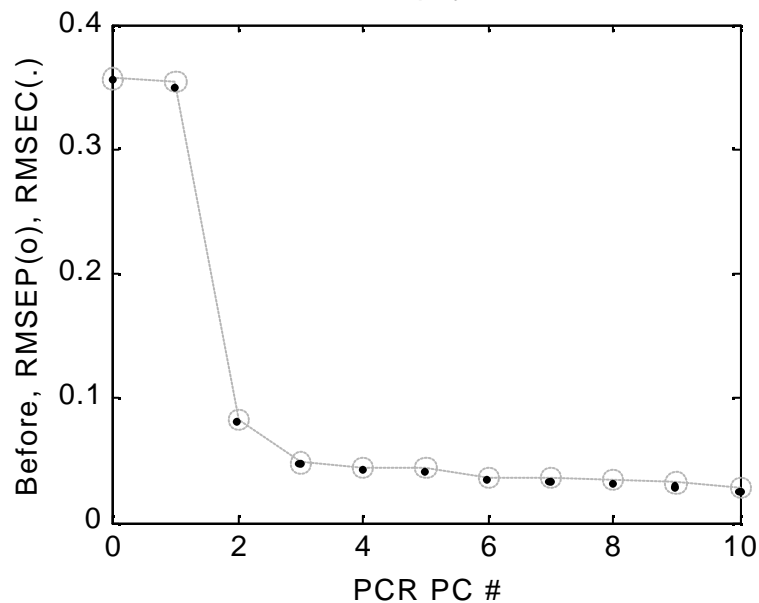
Output, DataCase=103, EMSC physical,default

Input, EMSC<sub>z</sub>.MAT

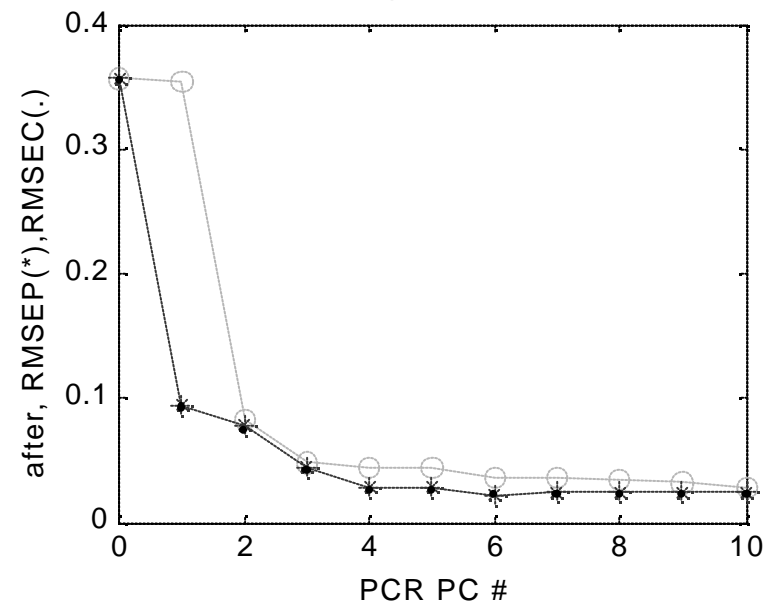
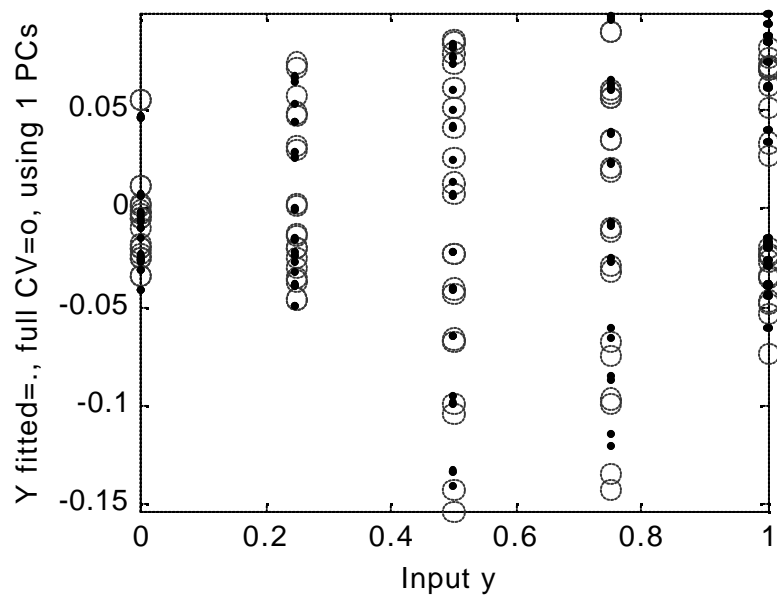
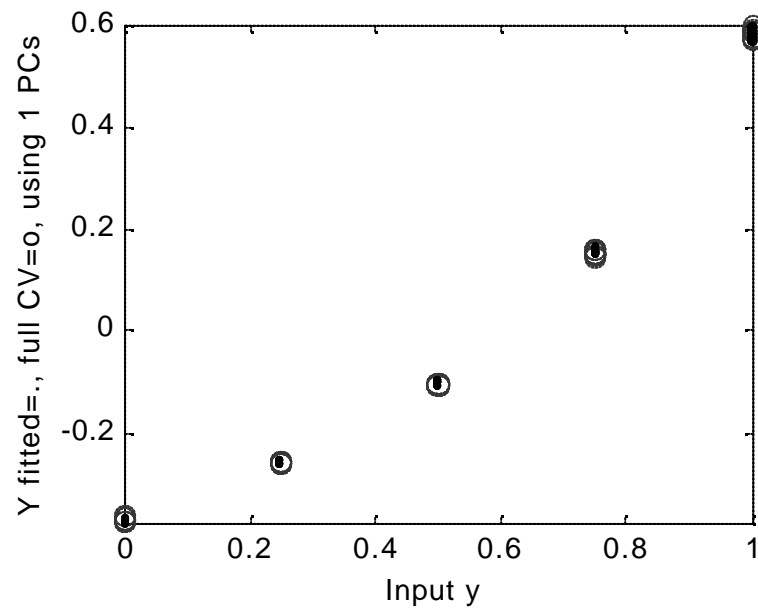
Output, DataCase=103, EMSC physical,default



DataCase=103 EMSC physical,default, before



after pre-treatment

Cal. for y from input Z,  $r_{CV}=0.046$ Cal. for y after EMSC/EISC,  $r_{CV}=0.964$ 

EMSC: Default physical model, no chemical model parameters,  
but **optimize the Reference spectrum  $\mathbf{m}$**

Instead of just using the mean spectrum:

$$\mathbf{m} = \bar{\mathbf{z}}'$$

Make bilinear model around the mean spectrum:

$$\mathbf{m} = t_{m,0} \bar{\mathbf{z}}' + t_{m,1} \mathbf{p}'_1 + t_{m,2} \mathbf{p}'_2 + t_{m,3} \mathbf{p}'_3$$

Estimate unknown parameters  $t_0, t_1, t_2, t_3$  by SIMPLEX Opt.,  
minimizing RMSEP(Y).

*(estimated leverage-corrected PCR, 1 PC)*

# EMSC Toolbox for Matlab:

<http://www.models.kvl.dk/source/EMSCtoolbox/index.asp>

Pre-defined default EMSC in the software:

...

```
elseif DataCase==(103)
```

```
    DataCaseName=' EMSC, opt. the Ref.spectrum, starting from the mean spectrum'
```

```
    OptPar=1
```

```
    ASearchDim=3
```

```
elseif ...
```

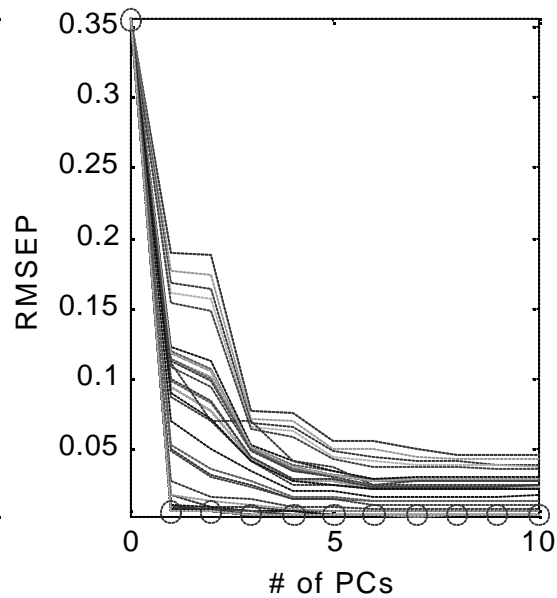
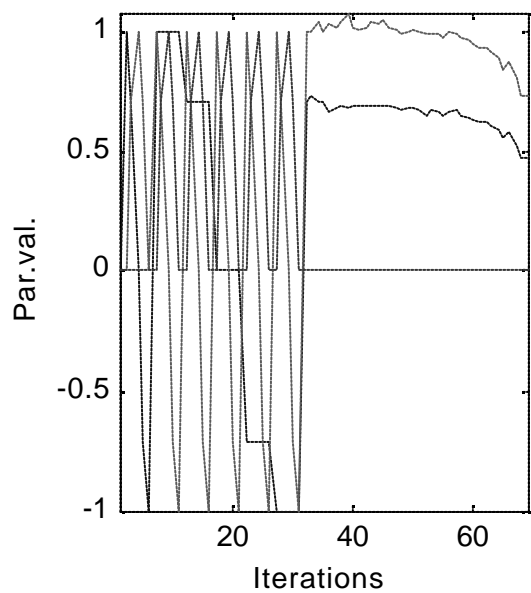
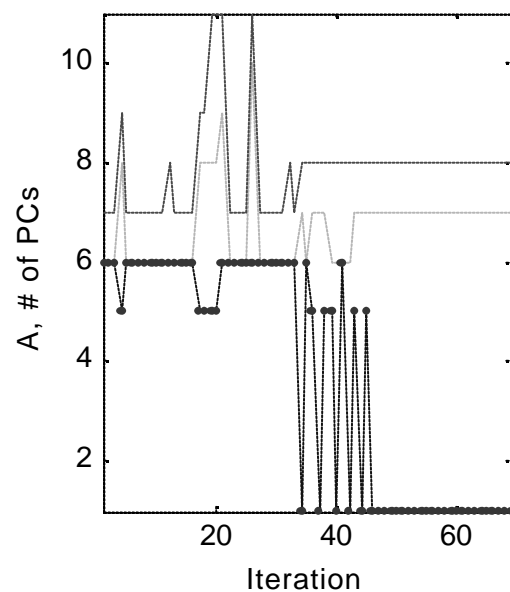
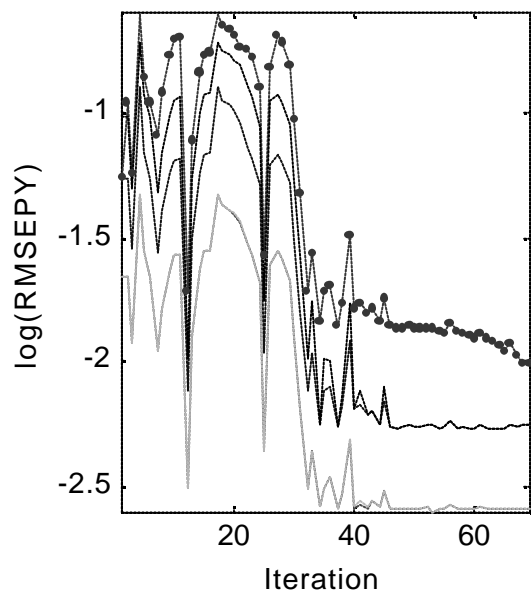
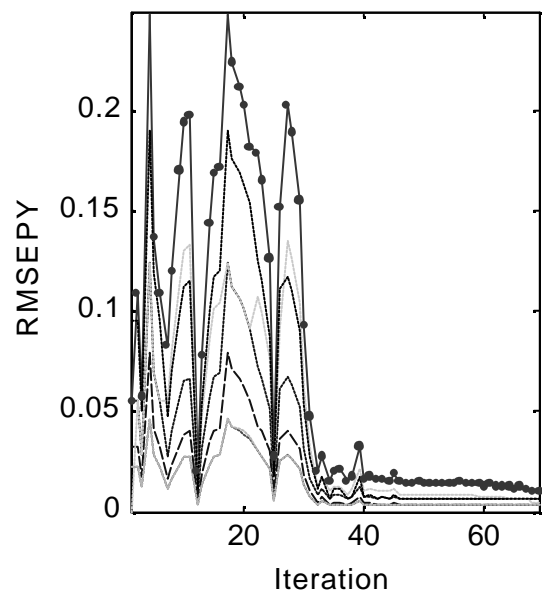
Running the program:

DataCase=? 151

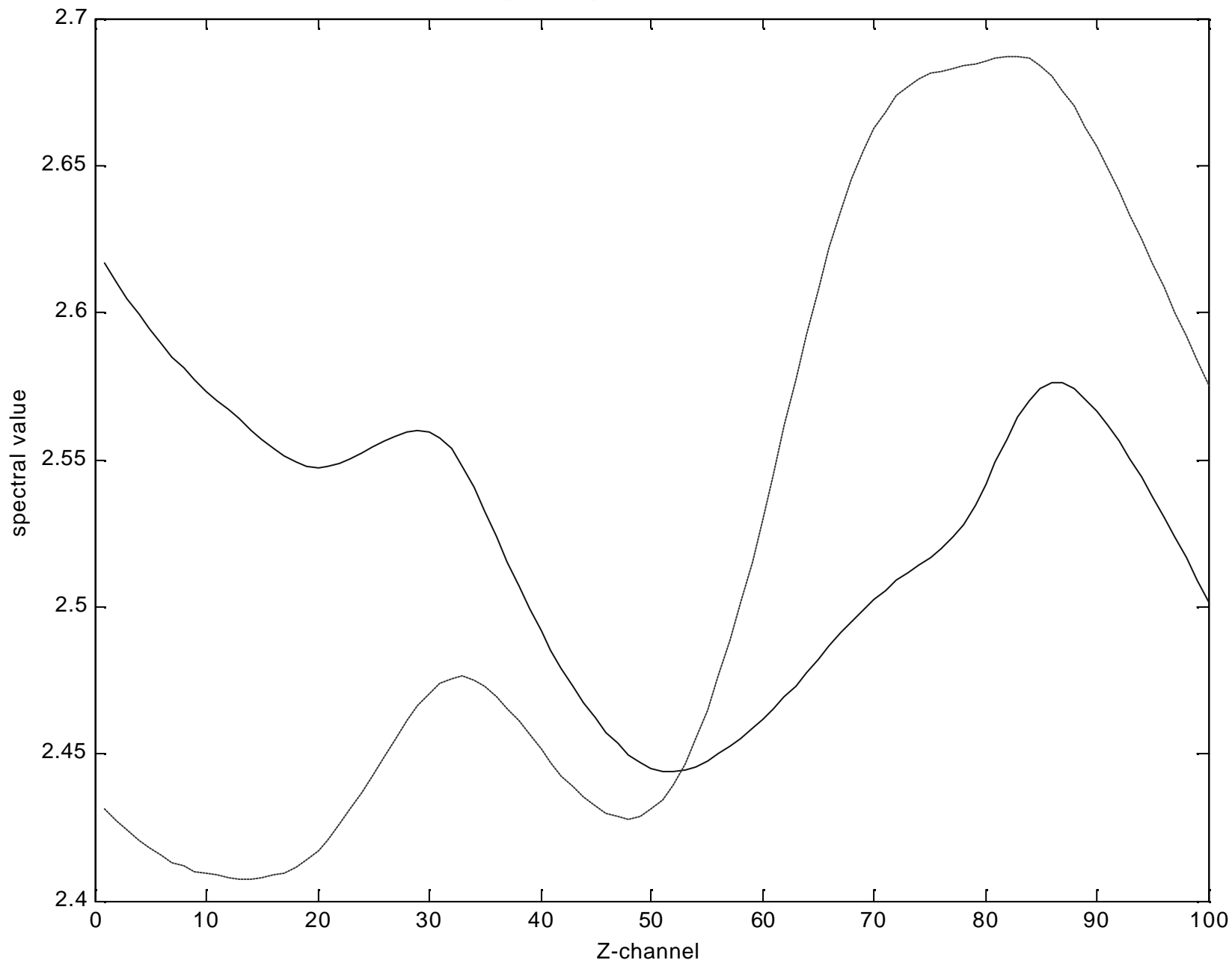
---



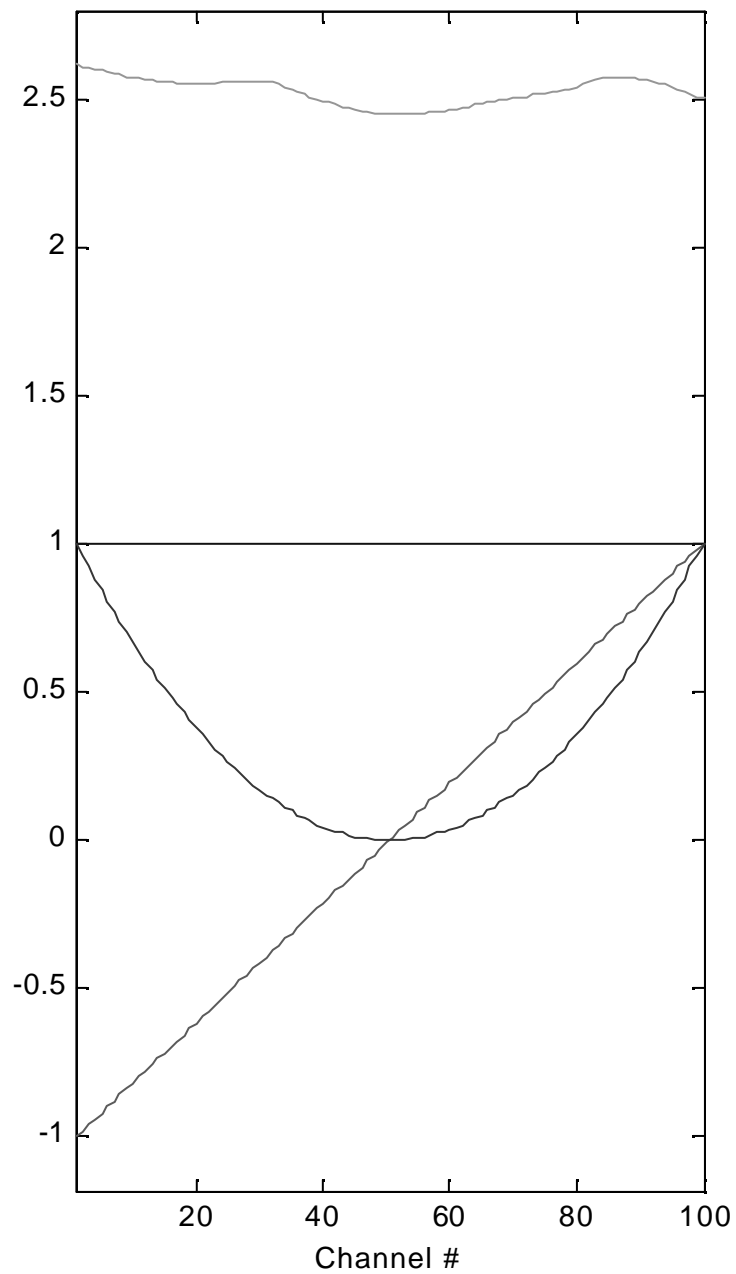
r=Crit., k=1PC, b:pun.opt, b--:A p.,g:opt, m:min Crit., k=1PC, b:pun.opt, g:opt, m:min A for: b:pun.opt, g:opt, m=min



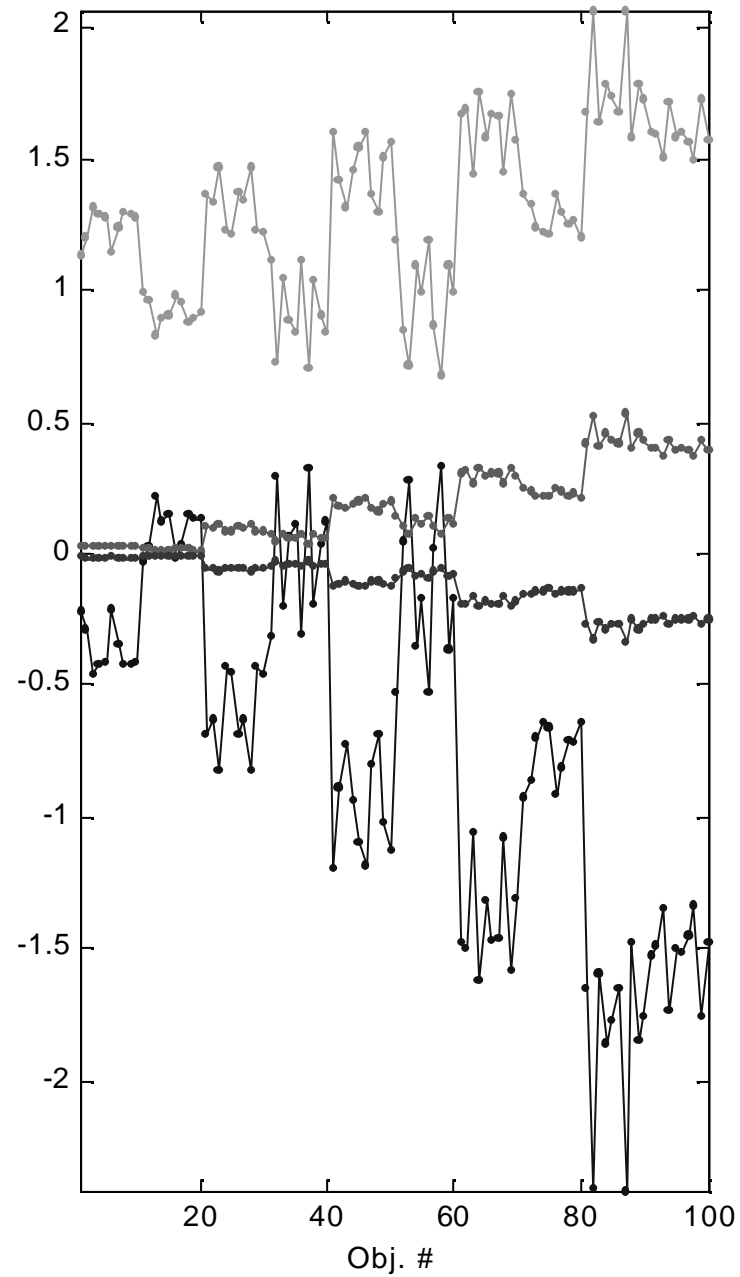
, Opt.Ref.spectrum , r...=its start

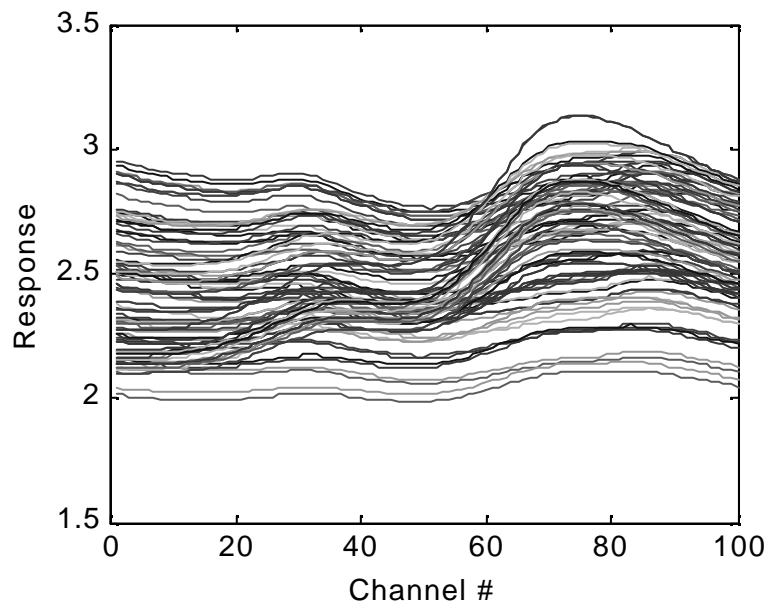


Model spectra

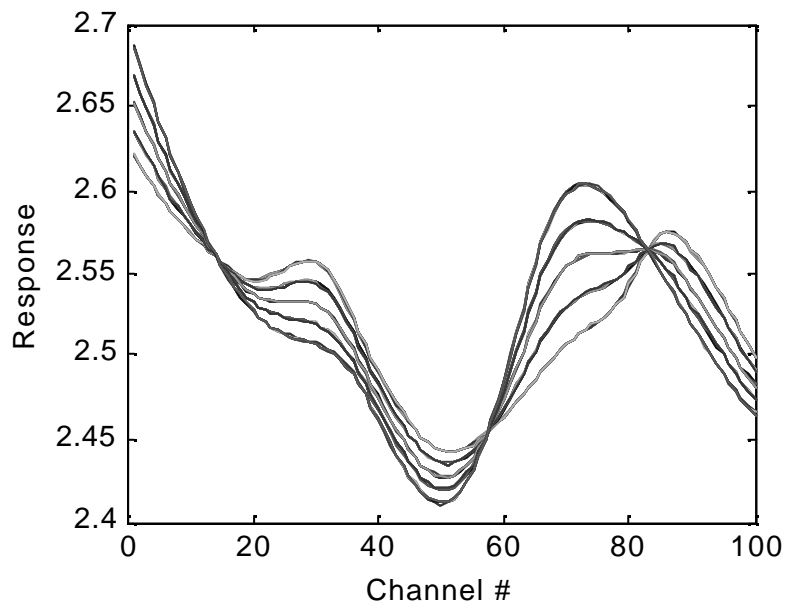
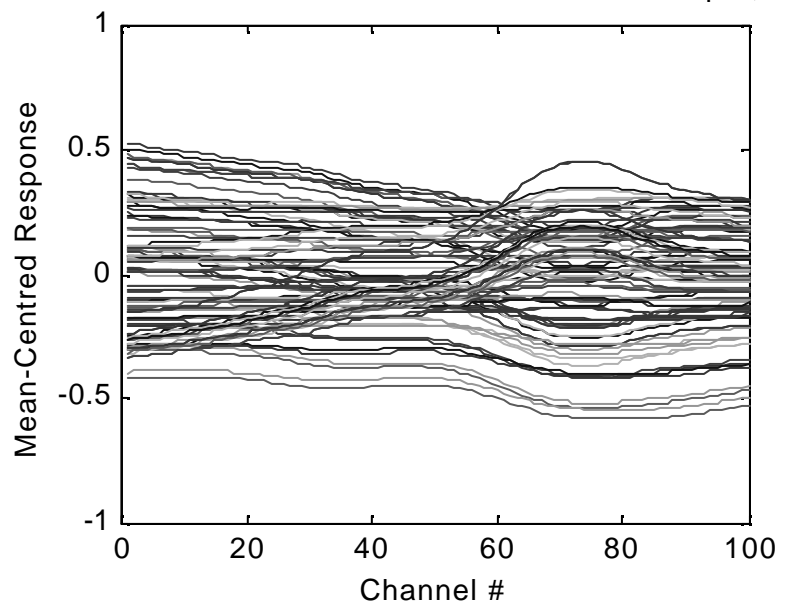


All parameter estimates together

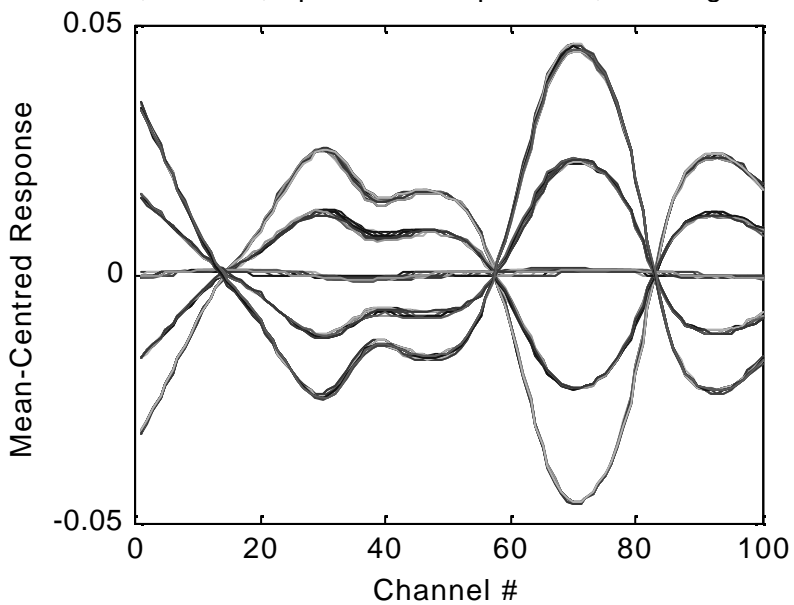


Input, EMSC<sub>Z</sub>.MAT

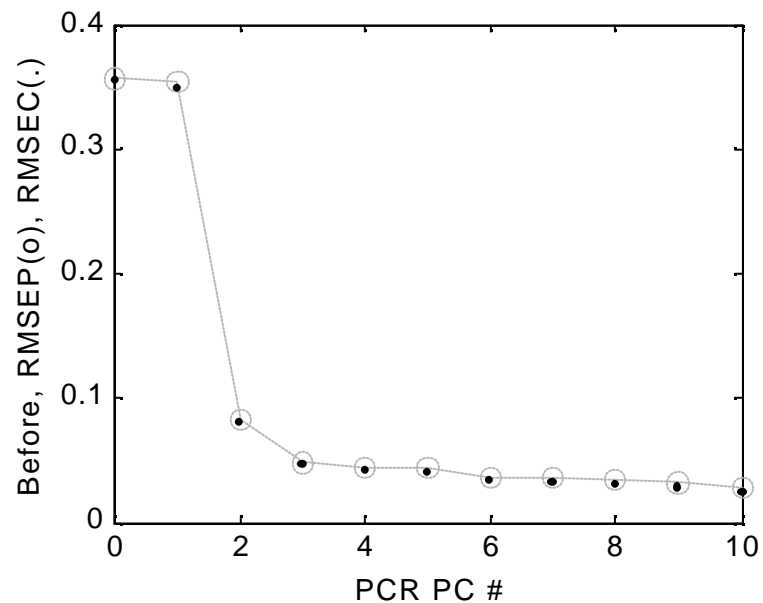
Output, DataCase=151, EMSC, opt. the Ref.spectrum, starting from the m

Input, EMSC<sub>Z</sub>.MAT

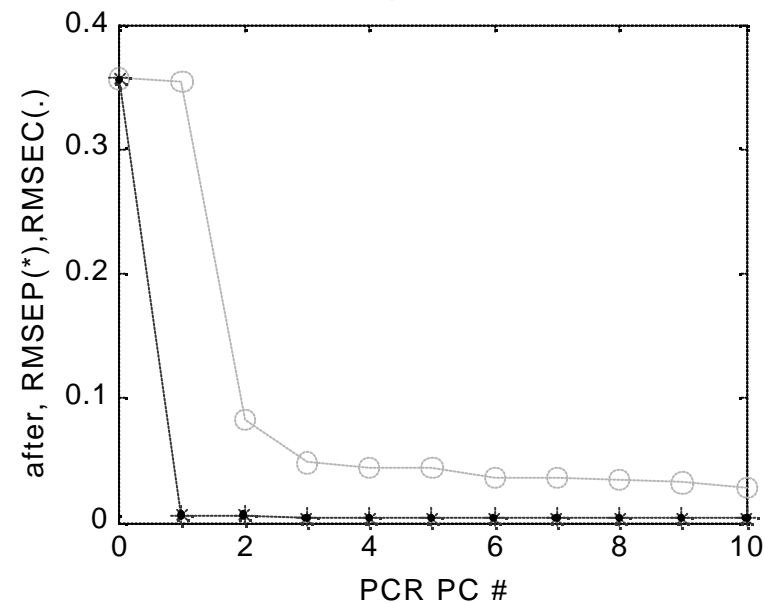
Output, DataCase=151, EMSC, opt. the Ref.spectrum, starting from the m



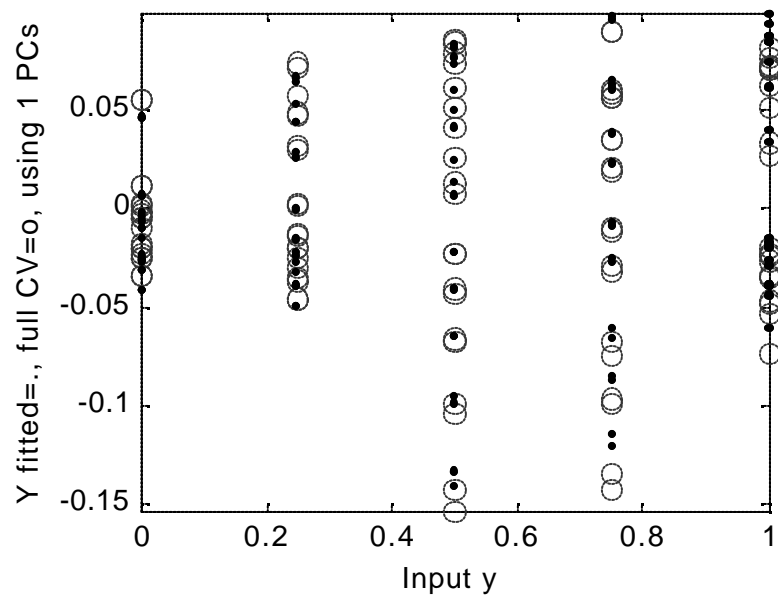
se=151 EMSC, opt. the Ref.spectrum, starting from the mean spectrum, before



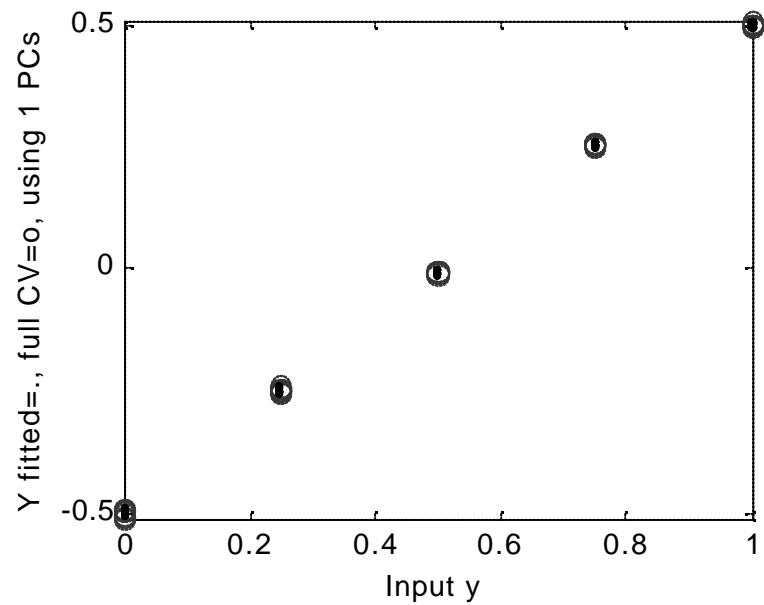
after pre-treatment

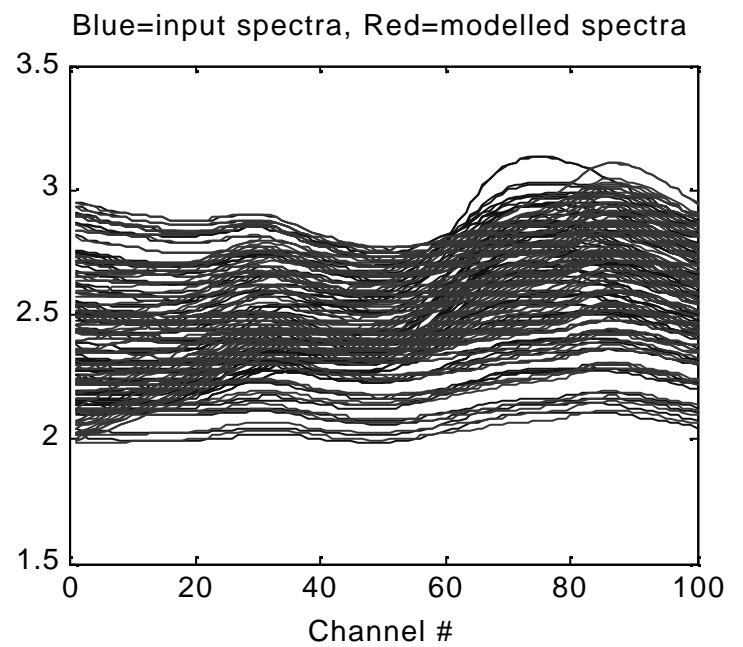
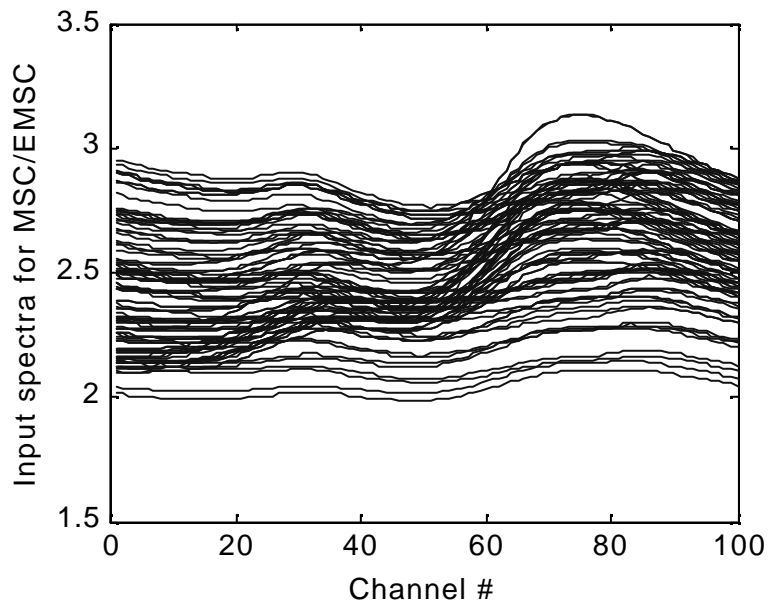


Cal. for y from input Z,  $r_{CV}=0.046$



Cal. for y after EMSC/EISC,  $r_{CV}=1$

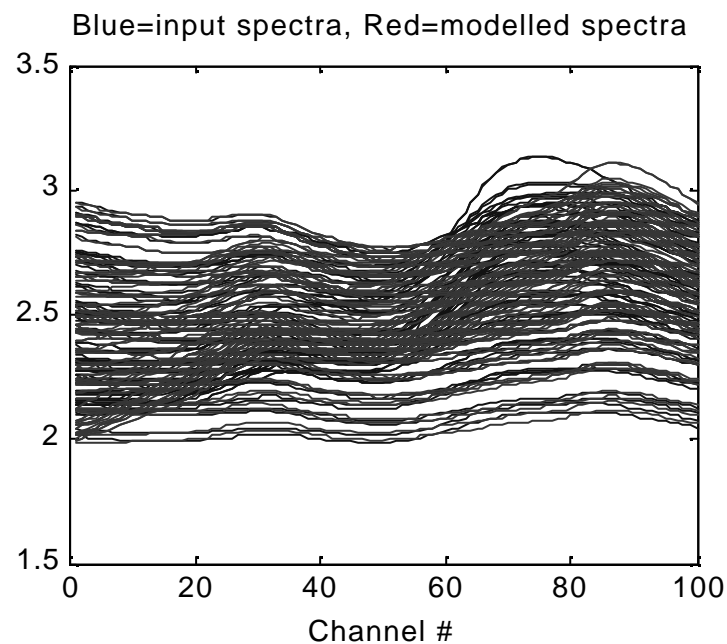
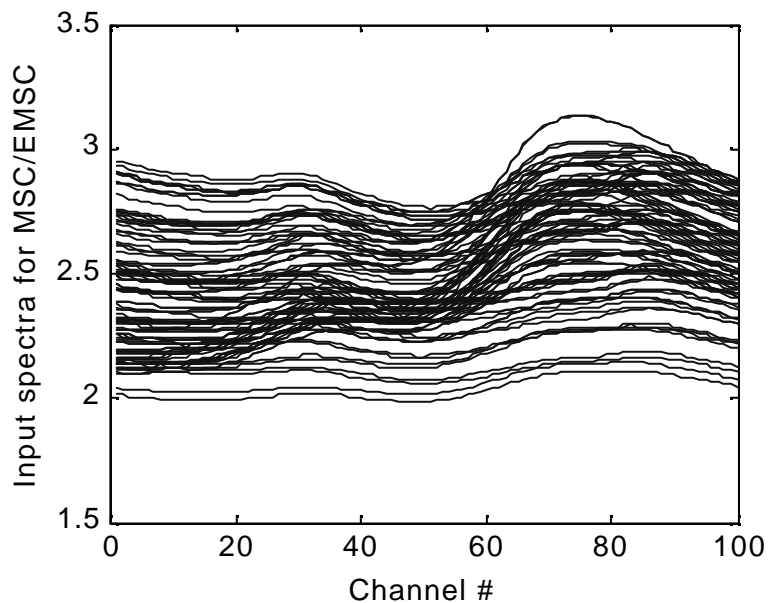




Input data:  $\mathbf{z}_i$

EMSC reconstruction :

$$\mathbf{z}_i = a_i \mathbf{1}' + b_i \mathbf{m}' + h_i \mathbf{k}' + d_i \mathbf{l} + e_i \mathbf{l}^2$$



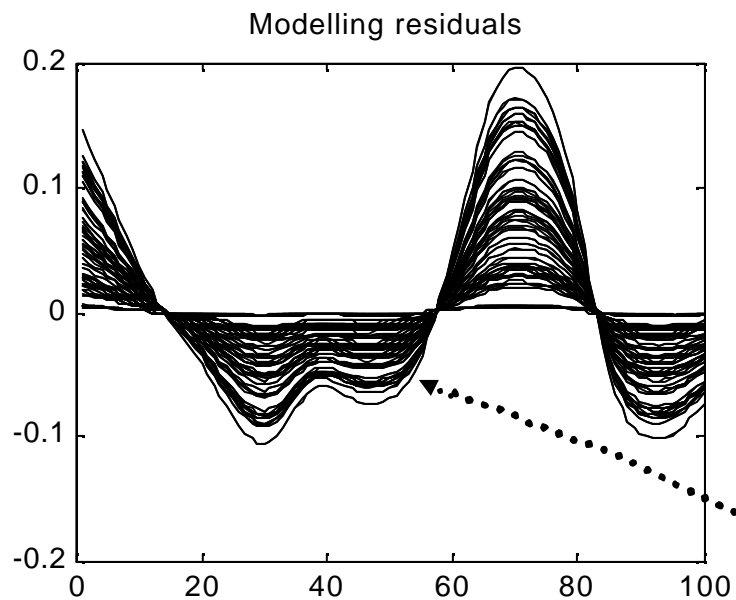
Input data:  $\mathbf{z}_i$

EMSC reconstruction :

$$\mathbf{z}_i = a_i \mathbf{1}' + b_i \mathbf{m}' + h_i \mathbf{k}' + d_i \mathbf{l} + e_i \mathbf{l}^2$$

Large un-modelled residuals  $e_i$  in model

$$\mathbf{z}_i = \mathbf{z}_i + e_i$$



EMSC Default physical model,  
+ a known “good” (analyte difference spectrum)  
+ a known “bad” spectrum (water)

### Extended ideal chemical model:

$$\mathbf{z}_{i,\text{chem}} = \mathbf{m}' + c_{i\text{Good}} \mathbf{K}_{\text{Good}}' + c_{i\text{Bad}} \mathbf{K}_{\text{Bad}}'$$

### The EMSC model of physical interferants:

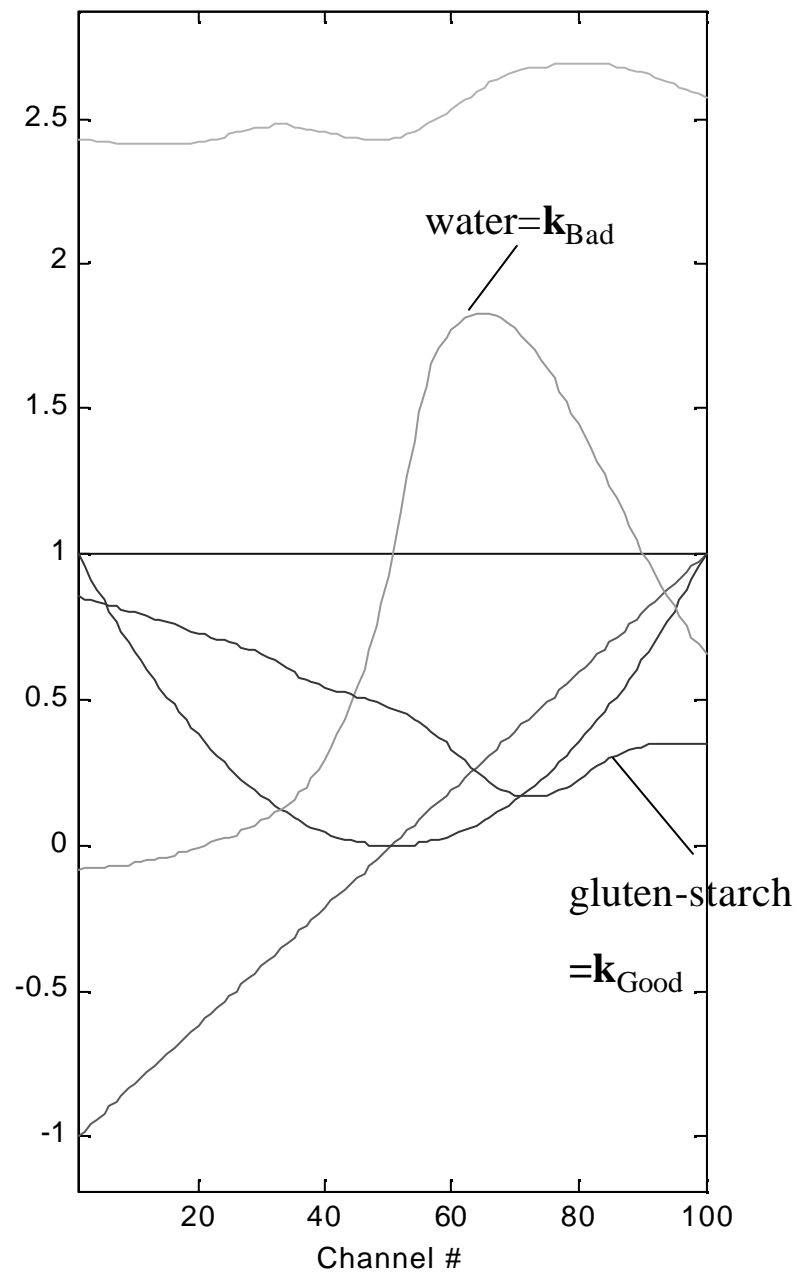
$$\mathbf{z}_i \approx a_i \mathbf{1}' + b_i \mathbf{z}_{i,\text{chem}} + d_i \mathbf{l} + e_i \mathbf{l}^2$$

$$\mathbf{z}_i = a_i \mathbf{1}' + b_i \mathbf{m}' + h_{i\text{Good}} \mathbf{k}_{\text{Good}}' + h_{i\text{Bad}} \mathbf{k}_{\text{Bad}}' + d_i \mathbf{l} + e_i \mathbf{l}^2 + \mathbf{e}_i$$

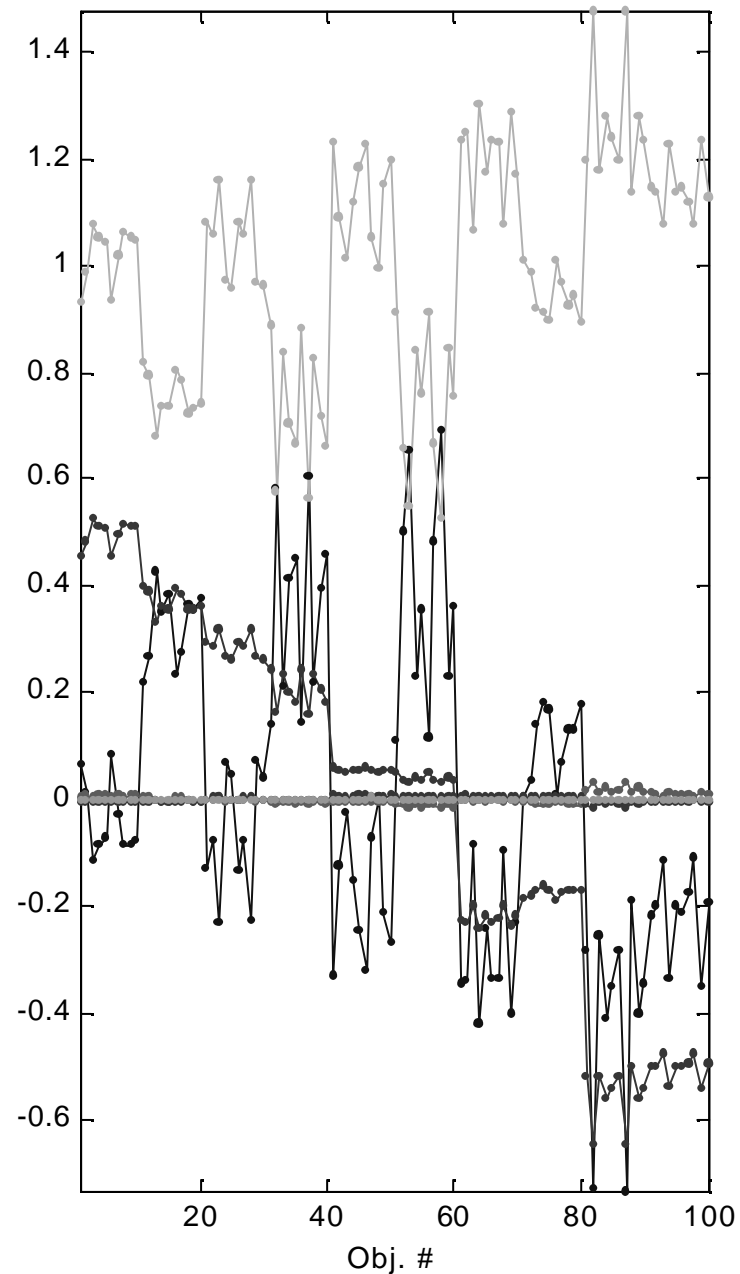
**EMSC corr.:**  $\mathbf{z}_{i,\text{corrected}} = (\mathbf{z}_i - a_i - h_{i\text{Bad}} \mathbf{k}_{\text{Bad}}' - d_i \mathbf{l} - e_i \mathbf{l}^2) / b_i$



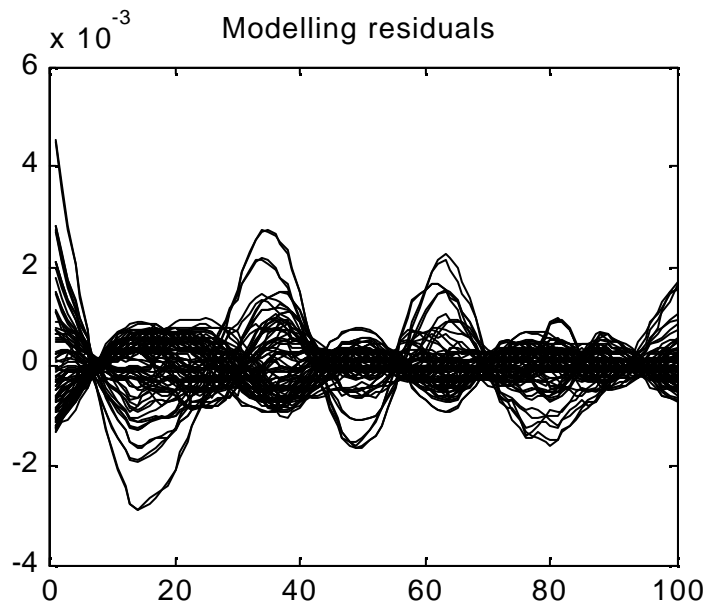
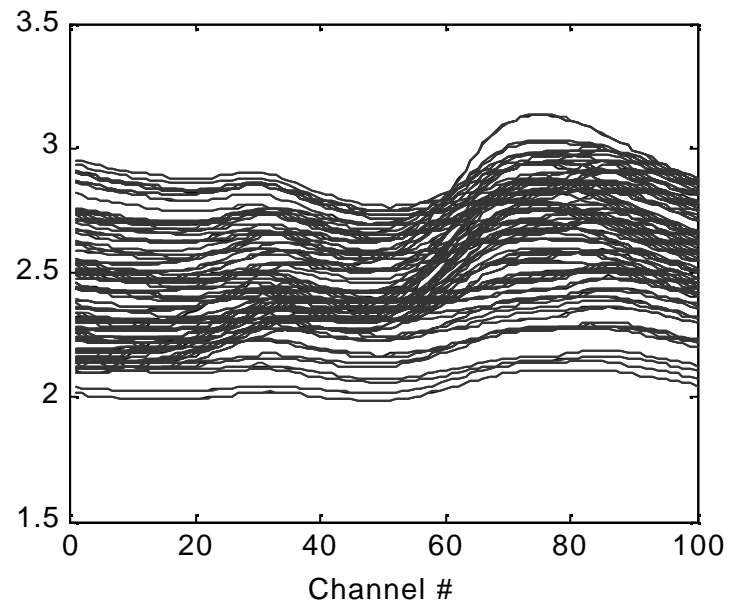
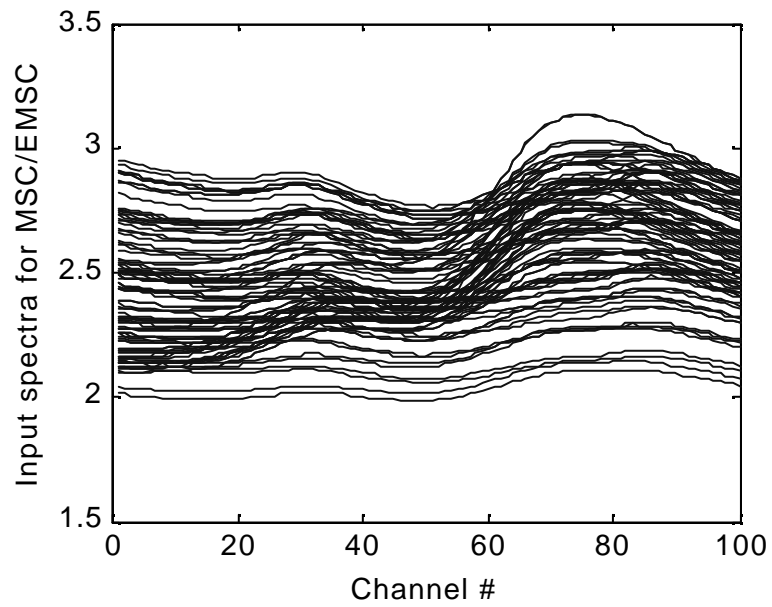
Model spectra



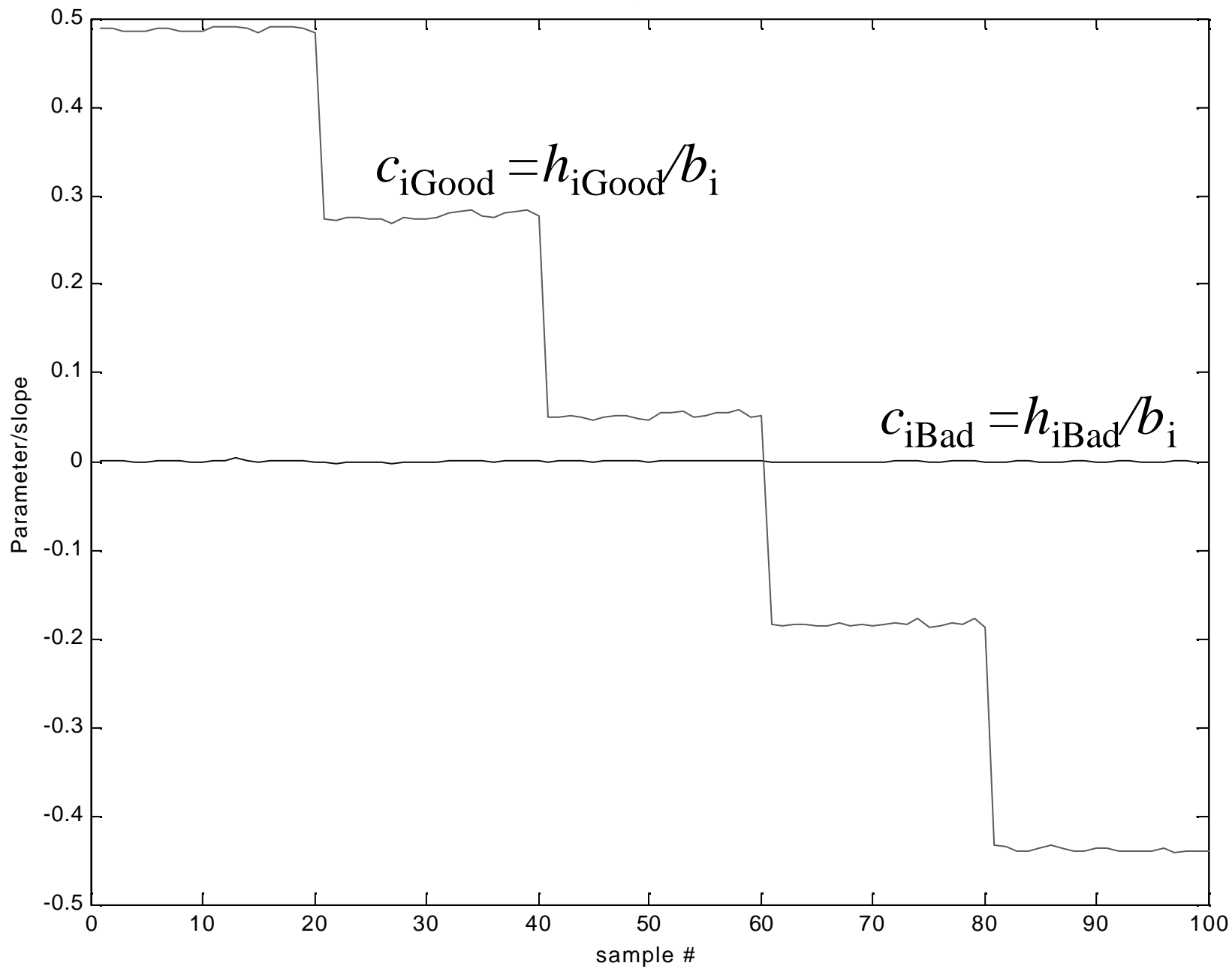
All parameter estimates together

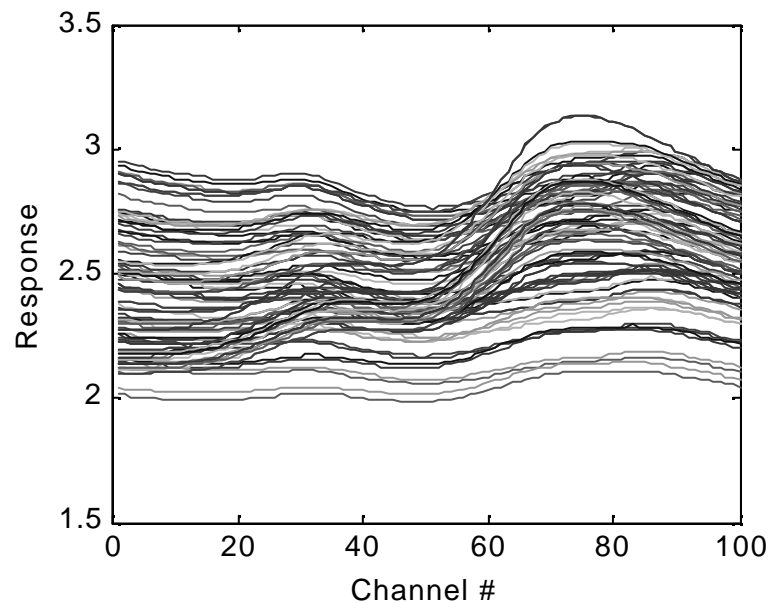


Blue=input spectra, Red=modelled spectra

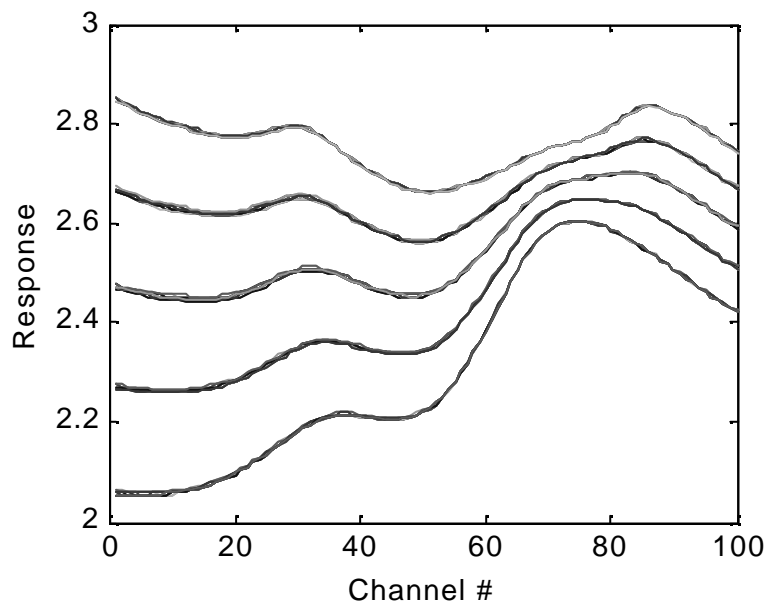
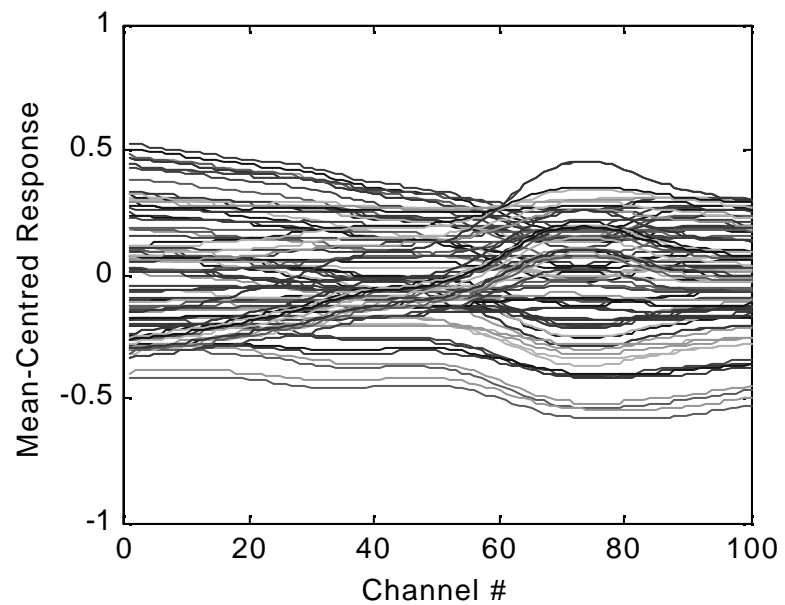


Corrected Chem. parameter estimates

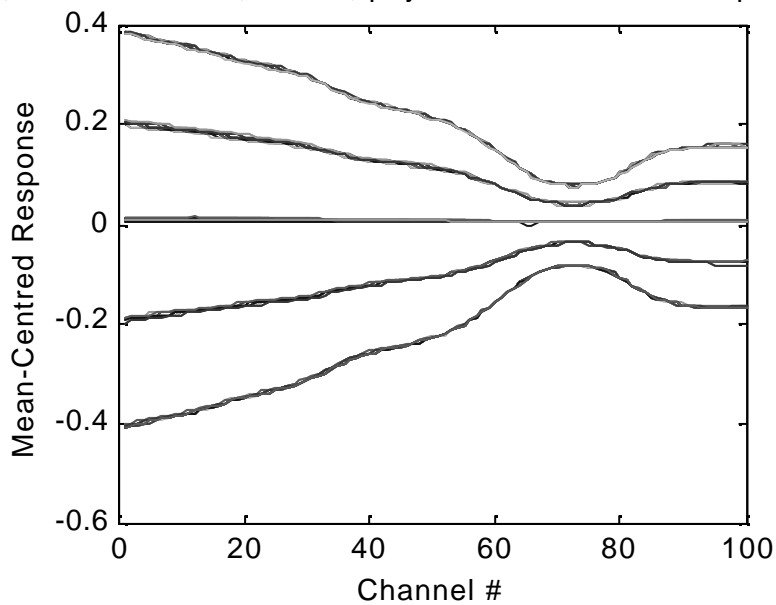


Input, EMSC<sub>Z</sub>.MAT

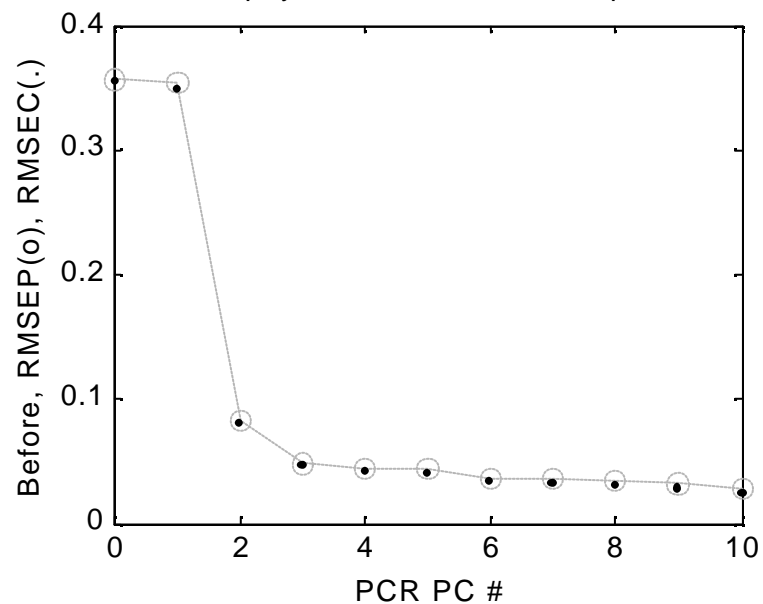
Output, DataCase=108, EMSC, physical &amp; Good &amp; Bad Spectra from

Input, EMSC<sub>Z</sub>.MAT

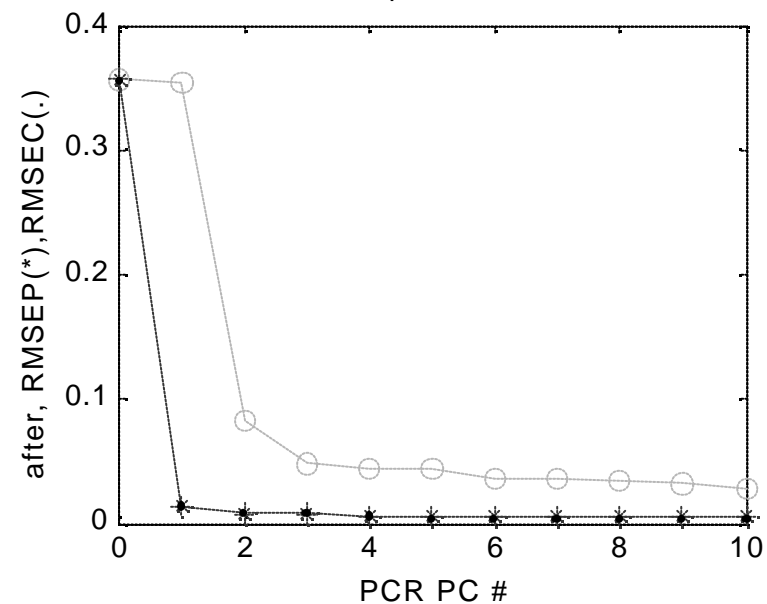
Output, DataCase=108, EMSC, physical &amp; Good &amp; Bad Spectra from



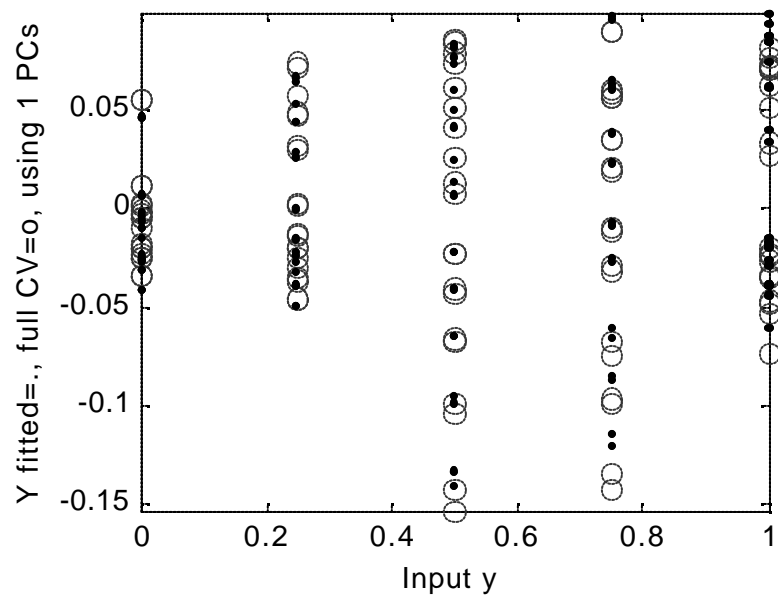
DataCase=108 EMSC, physical & Good & Bad Spectra from file, before



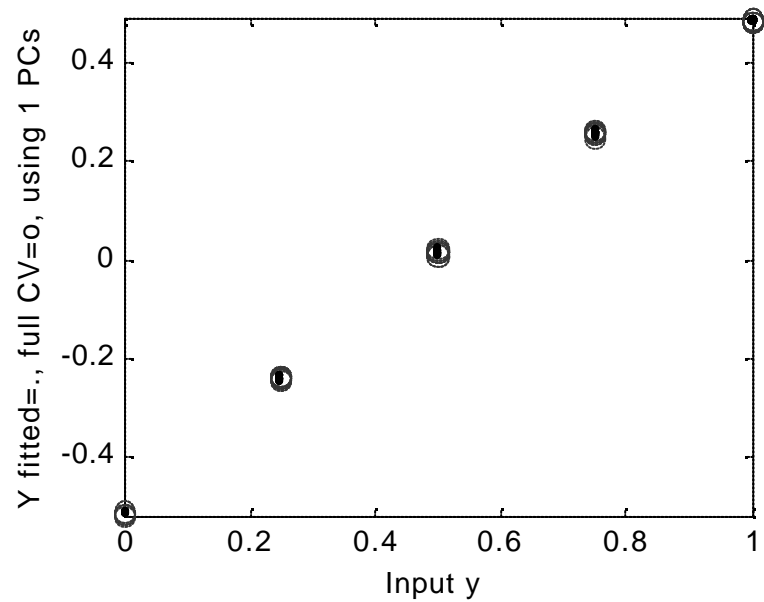
after pre-treatment



Cal. for y from input Z,  $r_{CV}=0.046$



Cal. for y after EMSC/EISC,  $r_{CV}=0.999$



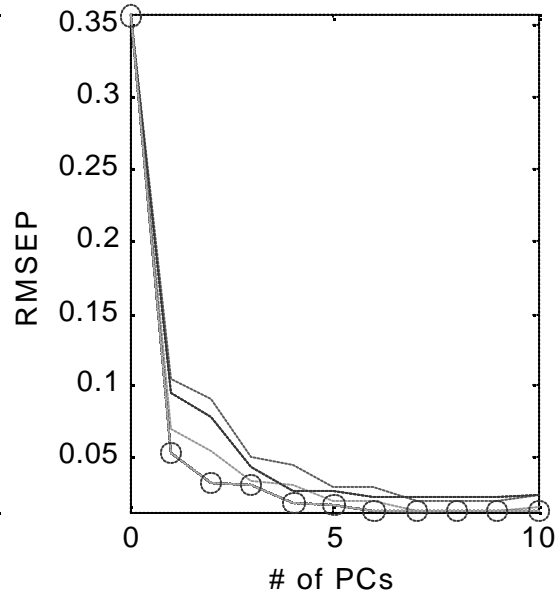
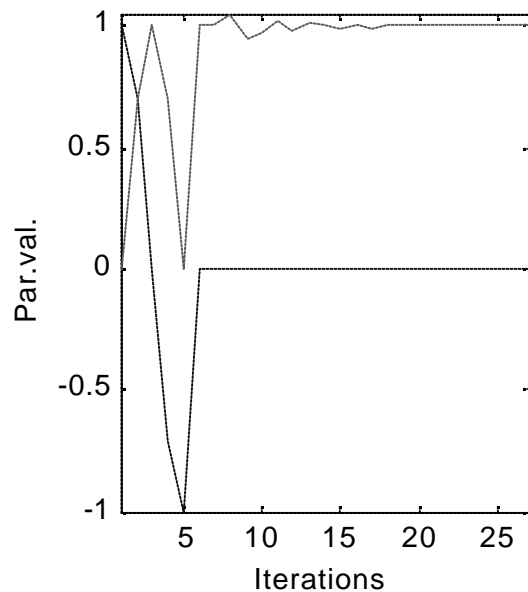
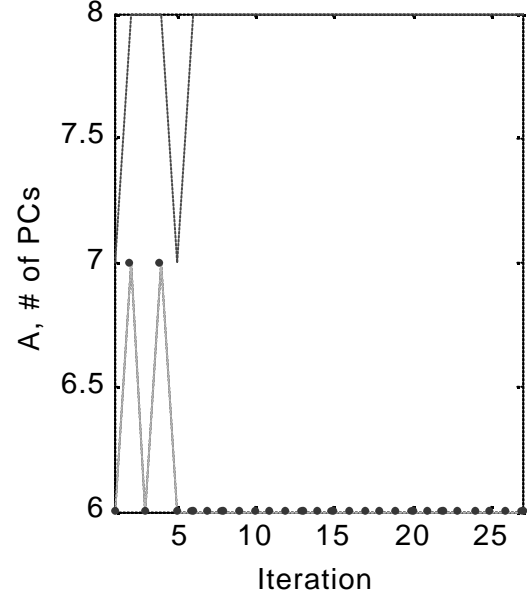
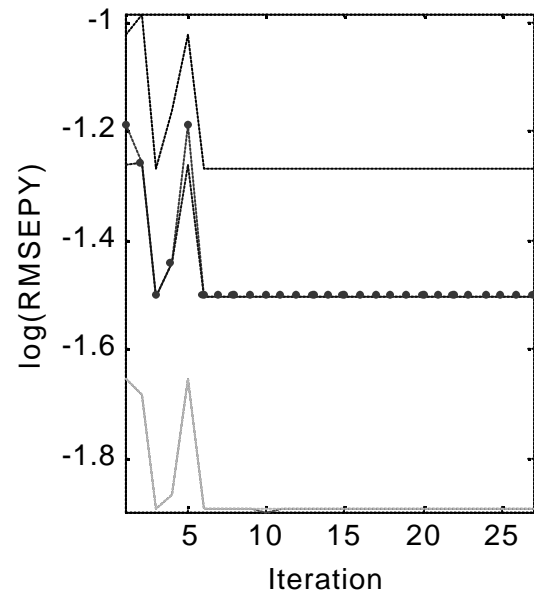
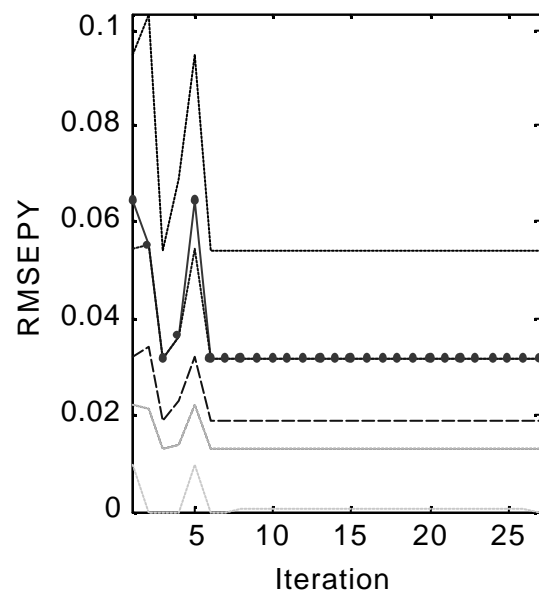
EMSC: Default physical model, no known good or bad spectra,  
but **automatically find and optimize**  
**an unknown “good” (analyte) spectrum**

EMSC: Default physical model, no known good or bad spectra,  
but **automatically find and optimize**  
**an unknown “good” (analyte) spectrum**  
*based on minimising  $RMSEP(Y)$*

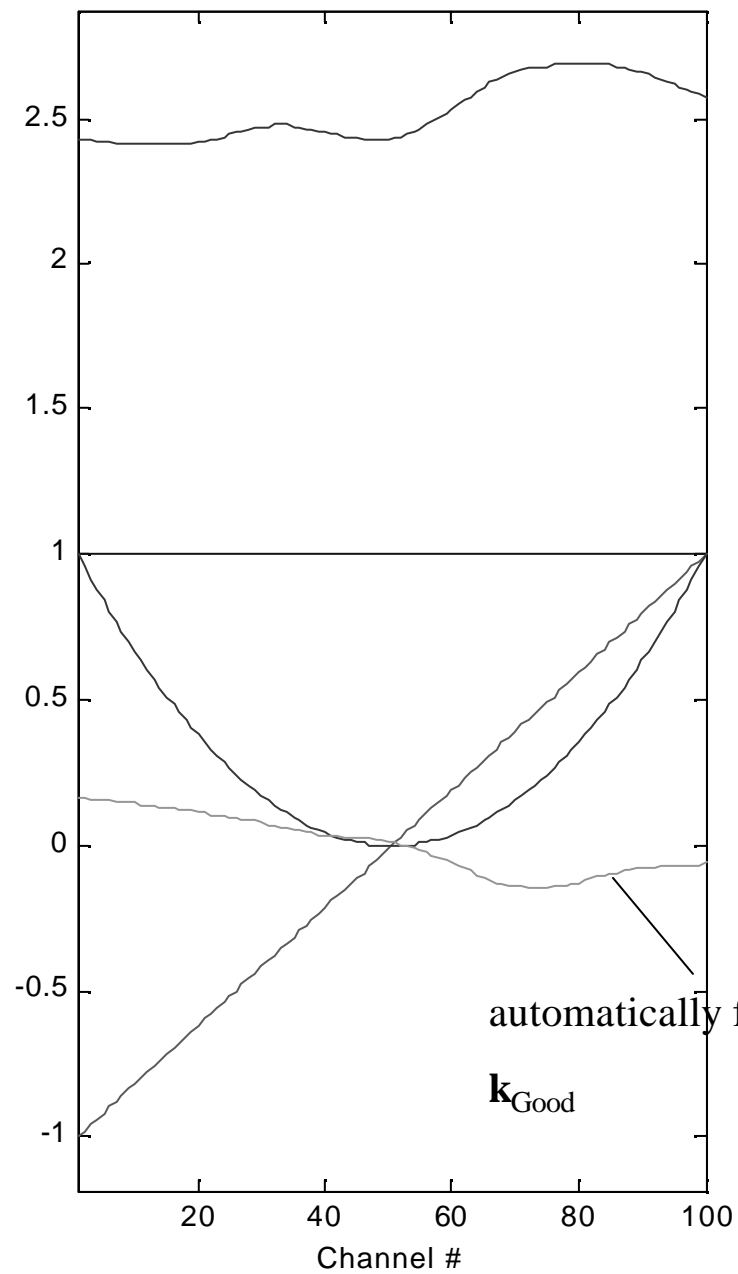
EMSC: Default physical model, no known good or bad spectra,  
but **automatically find and optimize**  
**an unknown “good” (analyte) spectrum**  
*based on minimising  $RMSEP(Y)$*   
*(estimated leverage-corrected PCR, 1 PC)*



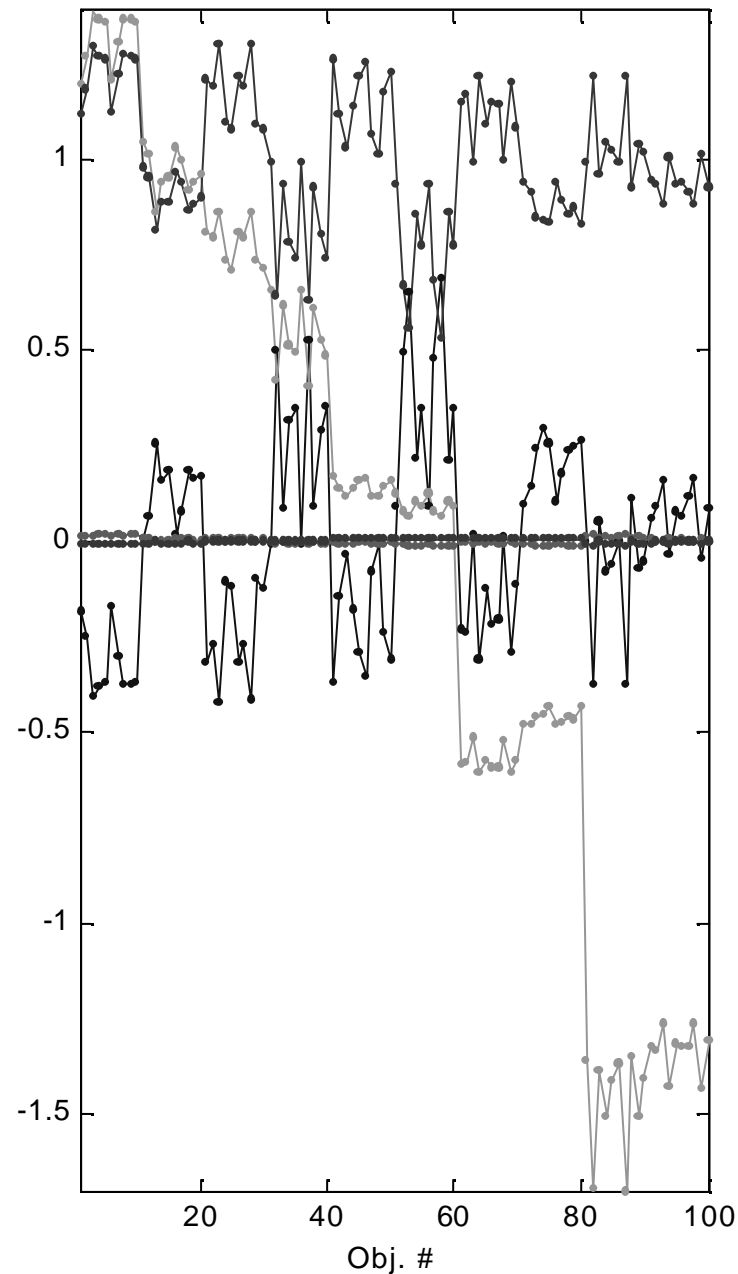
r=Crit., k=1PC, b:pun.opt, b--:A p.,g:opt, m:min  
r=Crit., k=1PC, b:pun.opt, g:opt, m=min  
A for: b:pun.opt, g=opt, m=min



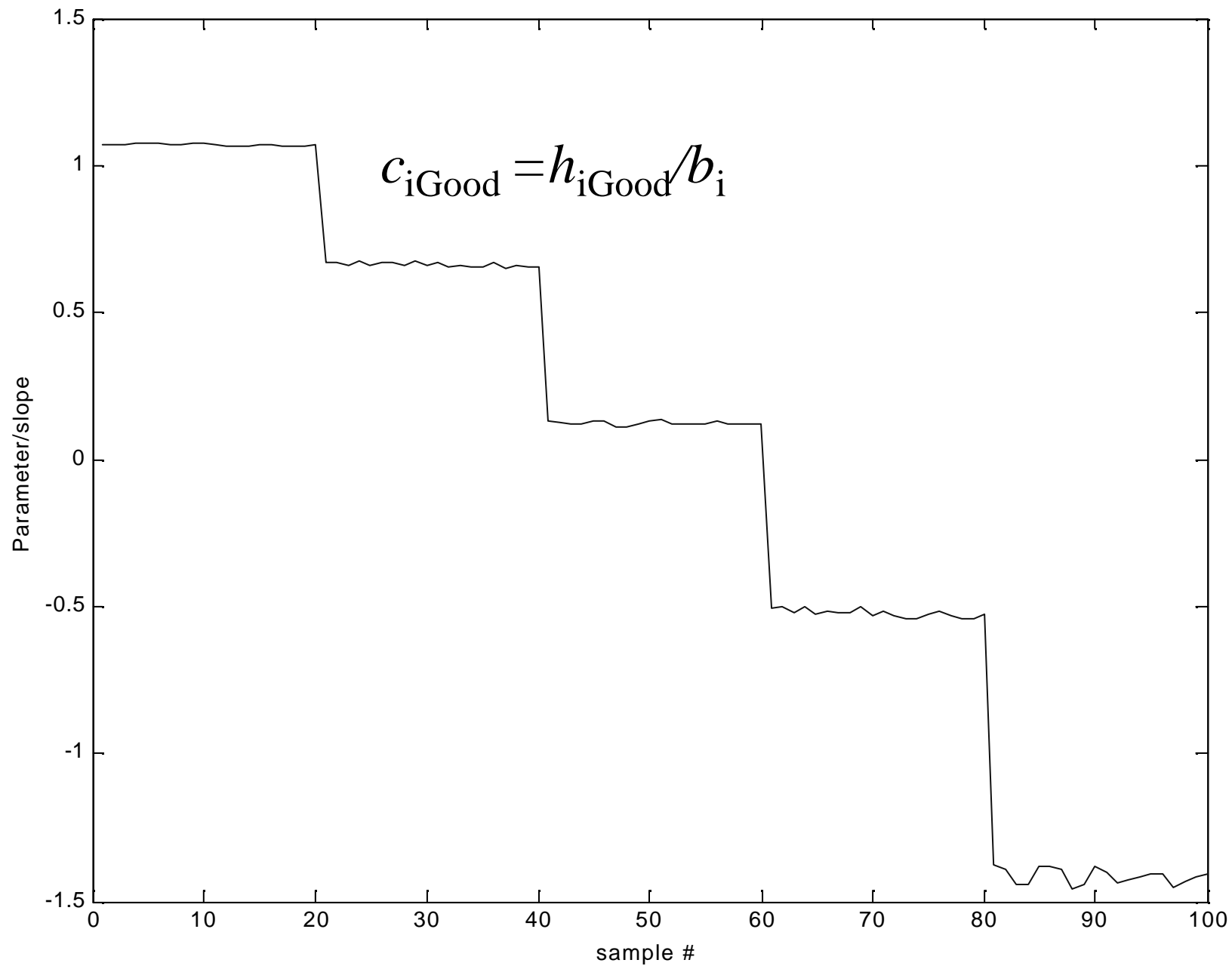
Model spectra

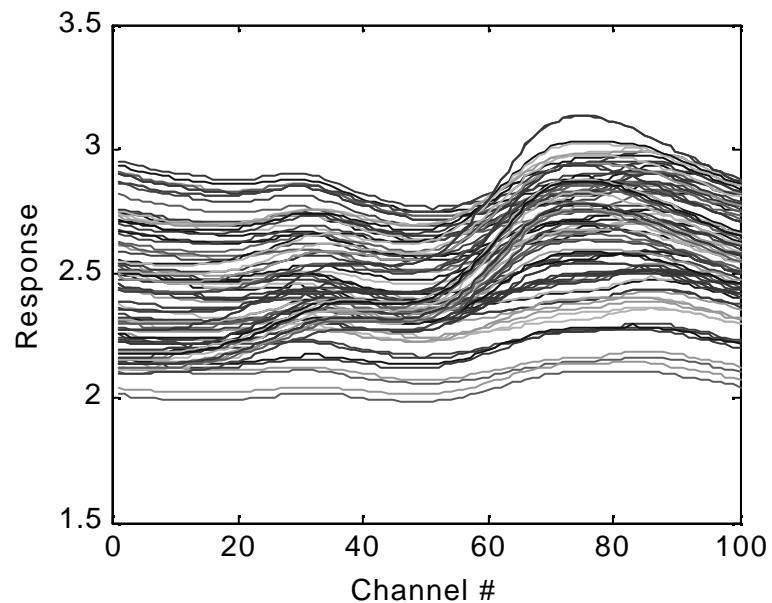


All parameter estimates together

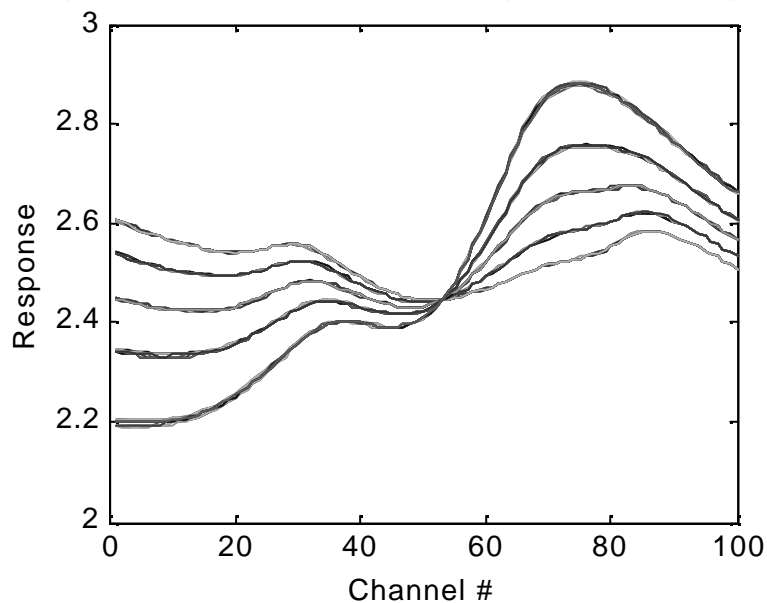
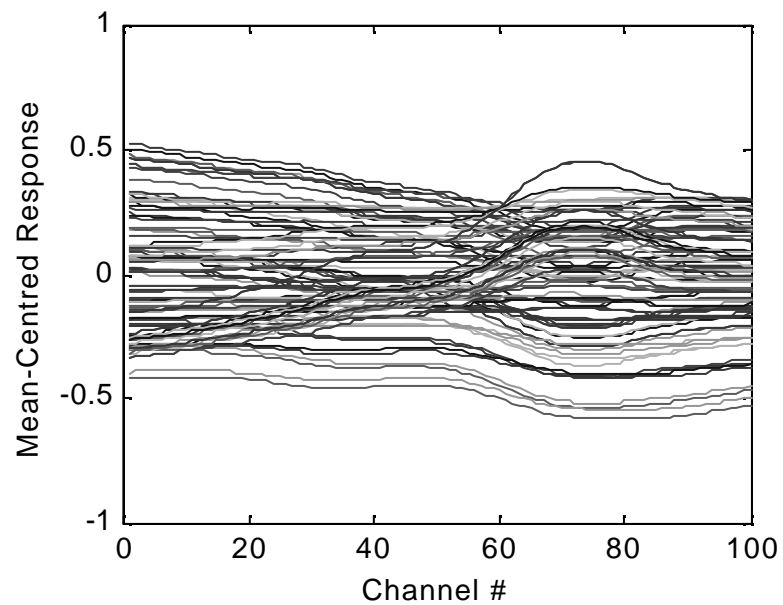


Corrected Chem. parameter estimates

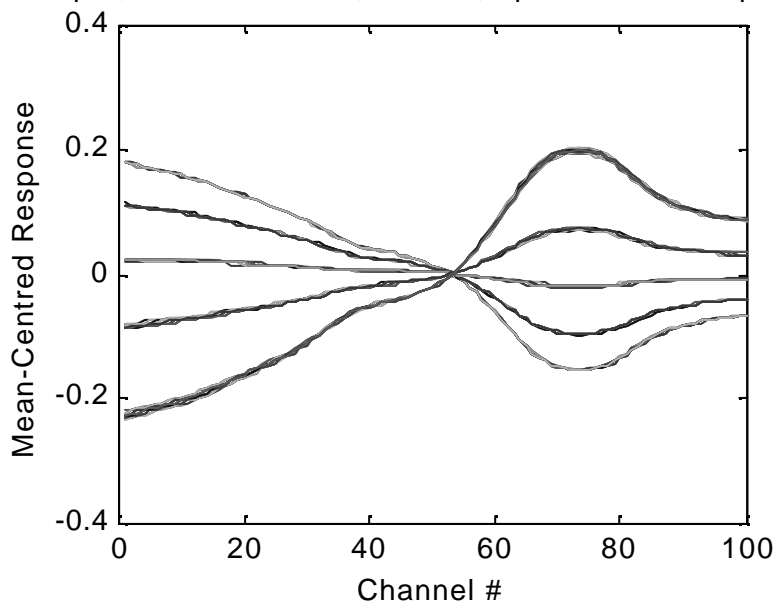


Input, EMSC<sub>z</sub>.MAT

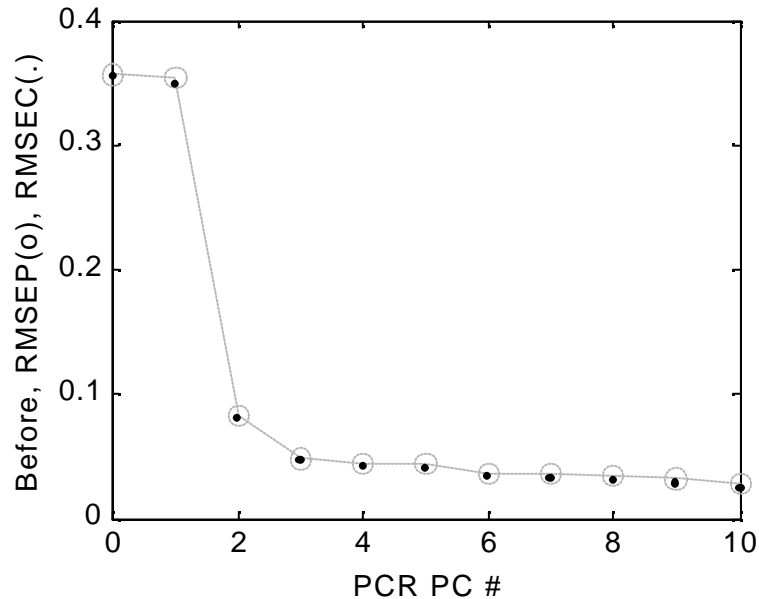
Output, DataCase=153, EMSC, opt. a Good component

Input, EMSC<sub>z</sub>.MAT

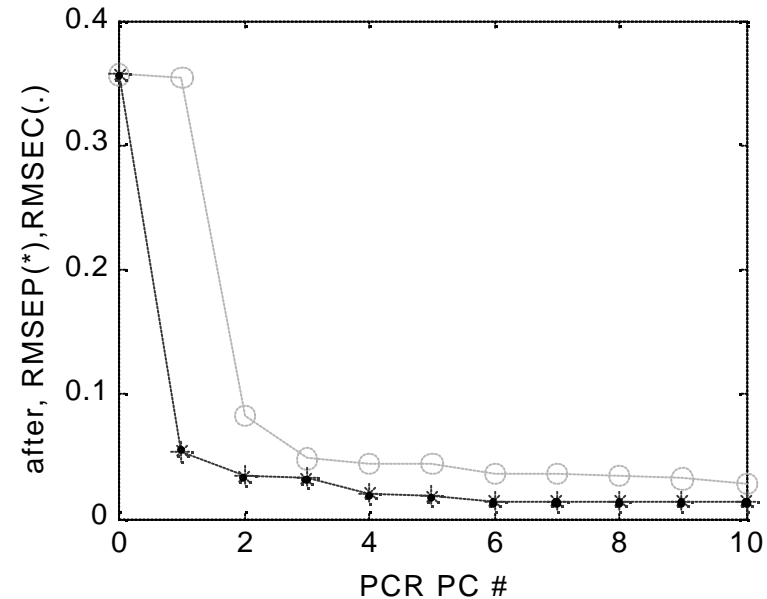
Output, DataCase=153, EMSC, opt. a Good component



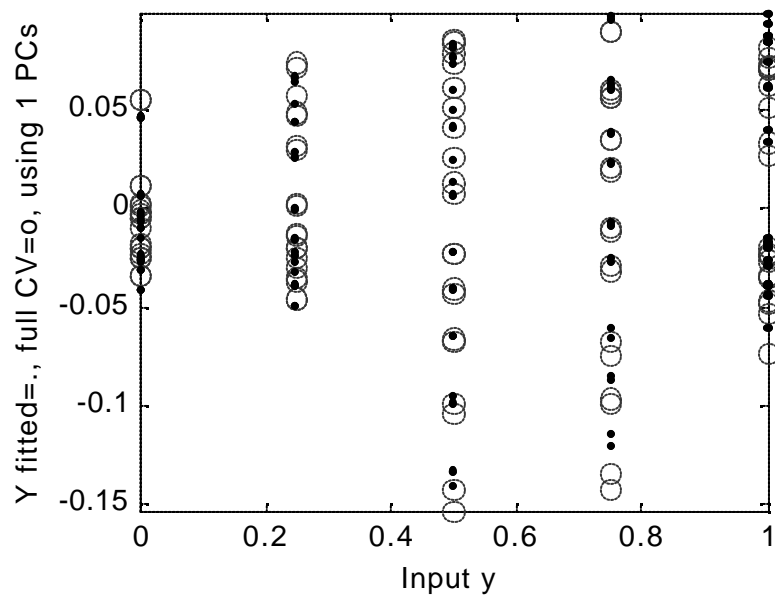
DataCase=153 EMSC, opt. a Good component, before



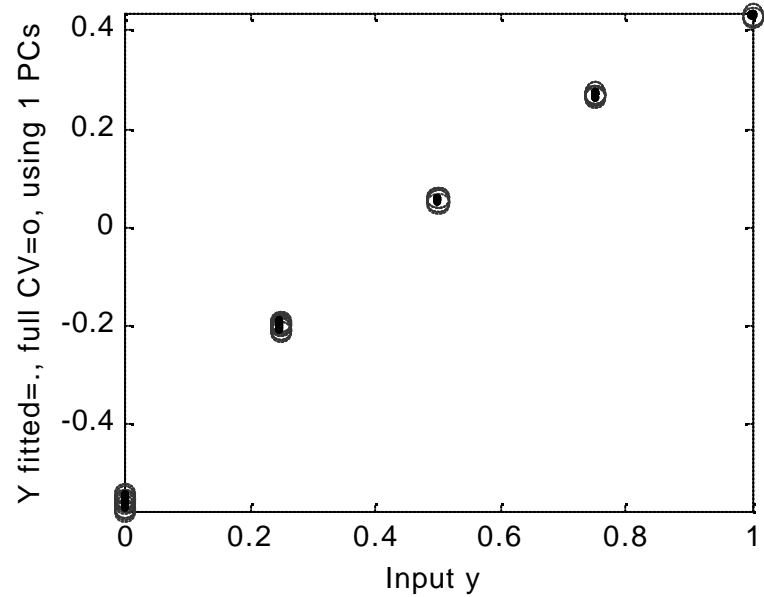
after pre-treatment



Cal. for y from input Z,  $r_{CV}=0.046$

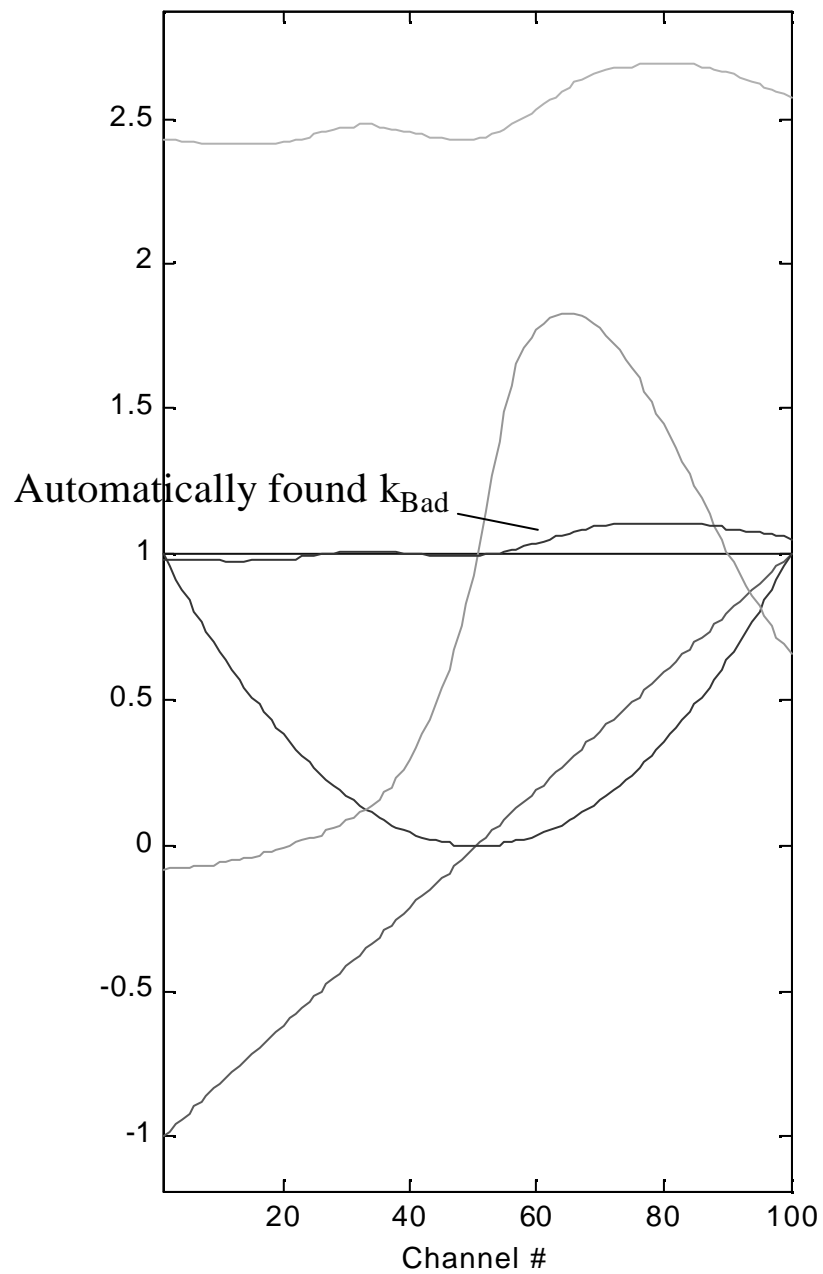


Cal. for y after EMSC/EISC,  $r_{CV}=0.988$

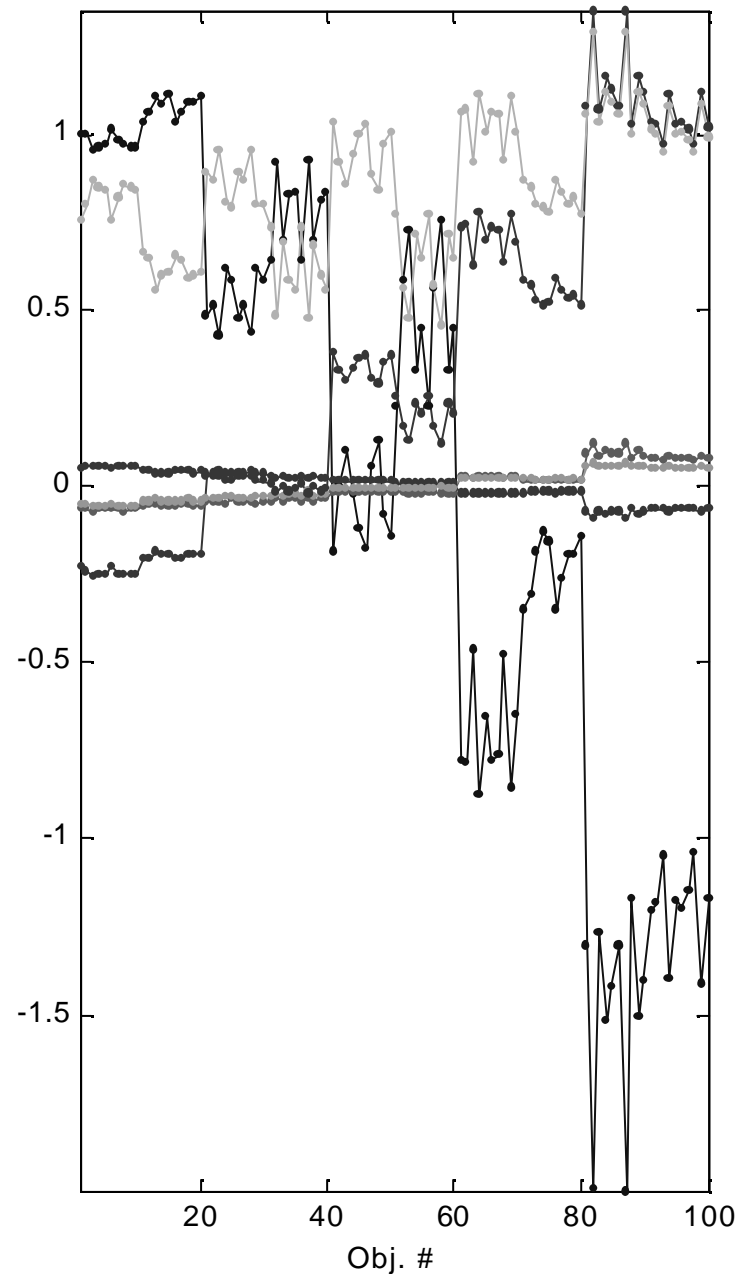


EMSC: Default physical model, input “bad” water spectrum  
but **find and optimized another “bad” spectrum**

Model spectra

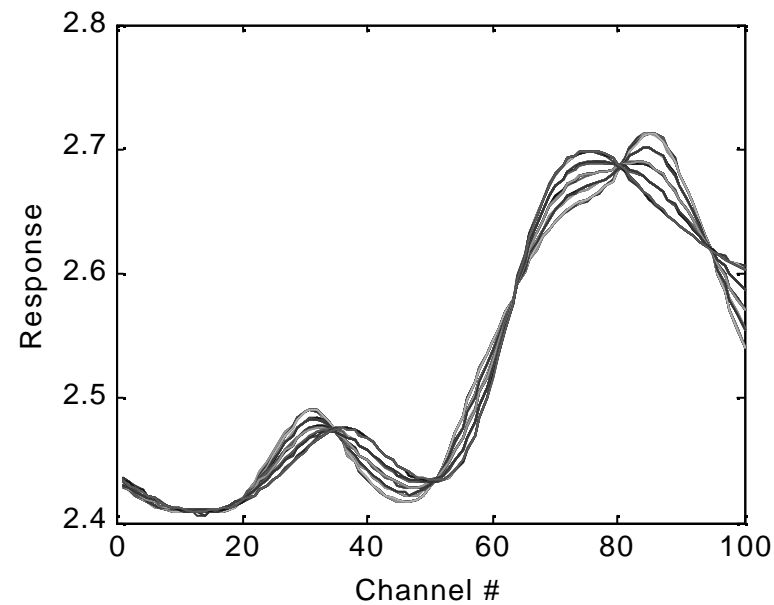
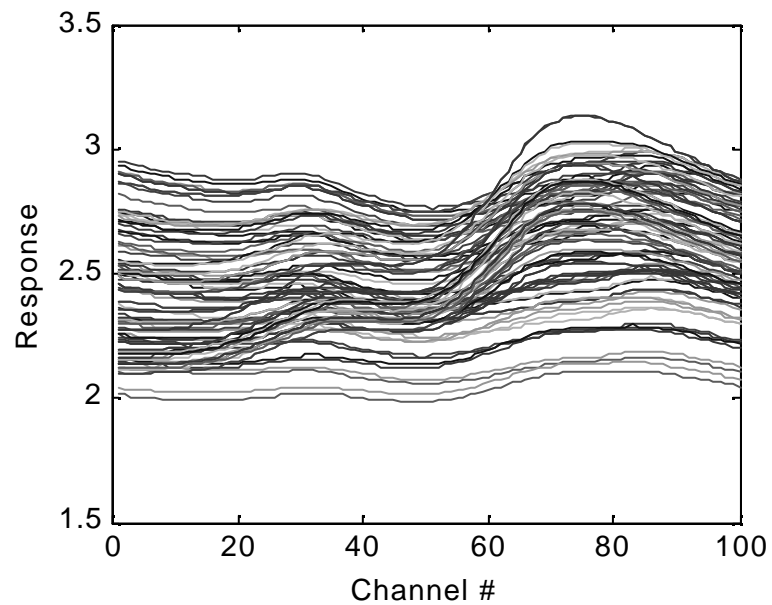


All parameter estimates together



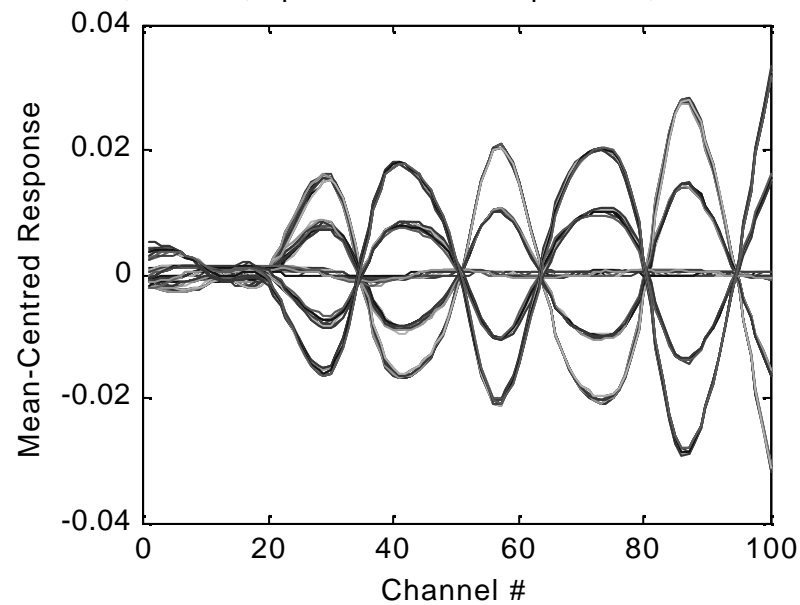
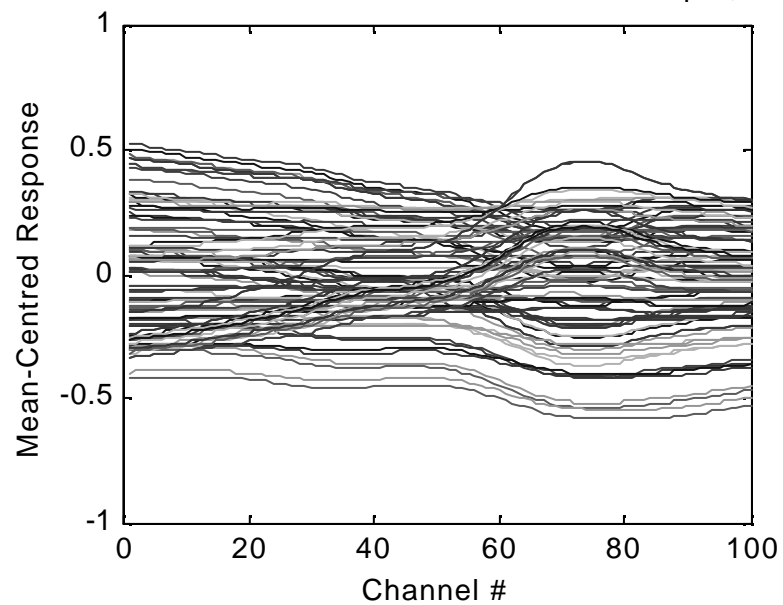
Input, EMSC<sub>Z</sub>.MAT

Output, DataCase=155, EMSC, opt.an extra Bad spectrum, in addition to inp



Input, EMSC<sub>Z</sub>.MAT

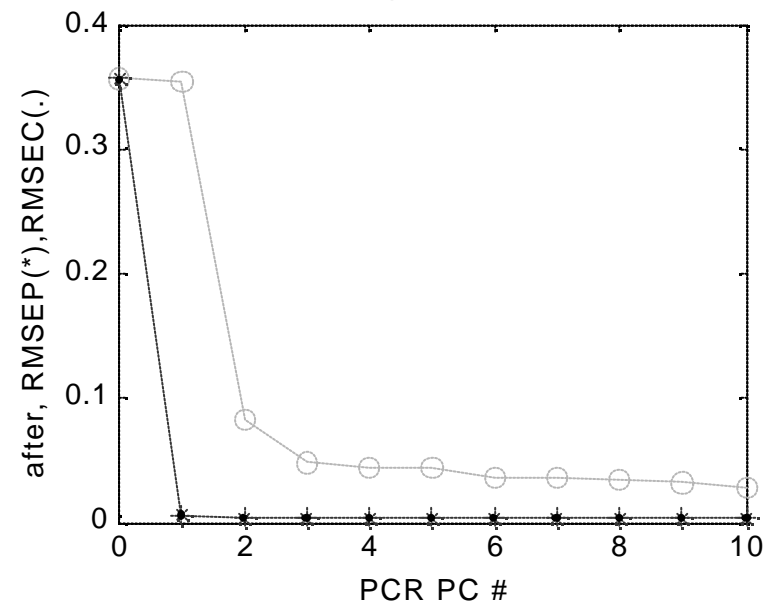
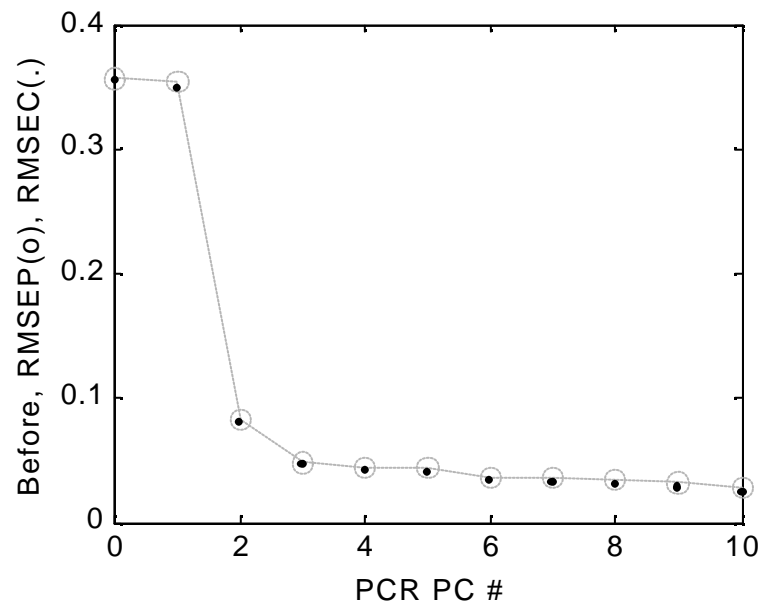
Output, DataCase=155, EMSC, opt.an extra Bad spectrum, in addition to inp



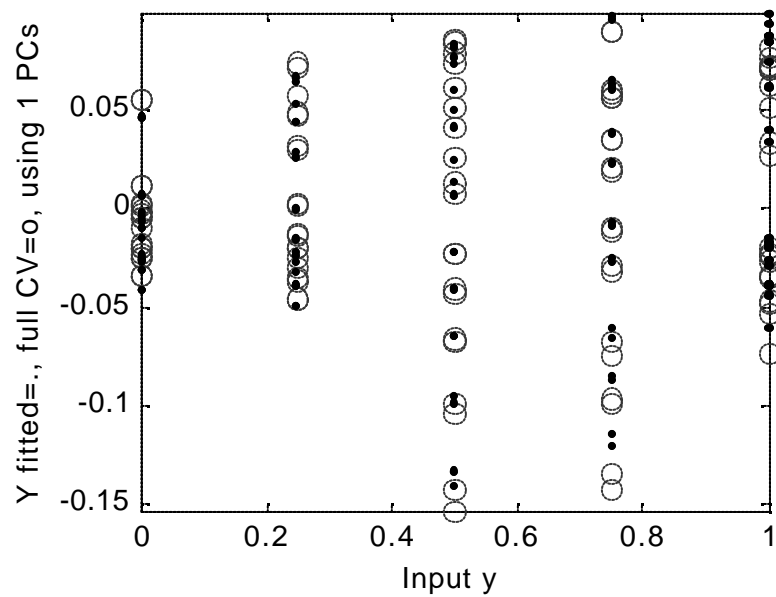


=155 EMSC, opt.an extra Bad spectrum, in addition to input BadSpectra, before

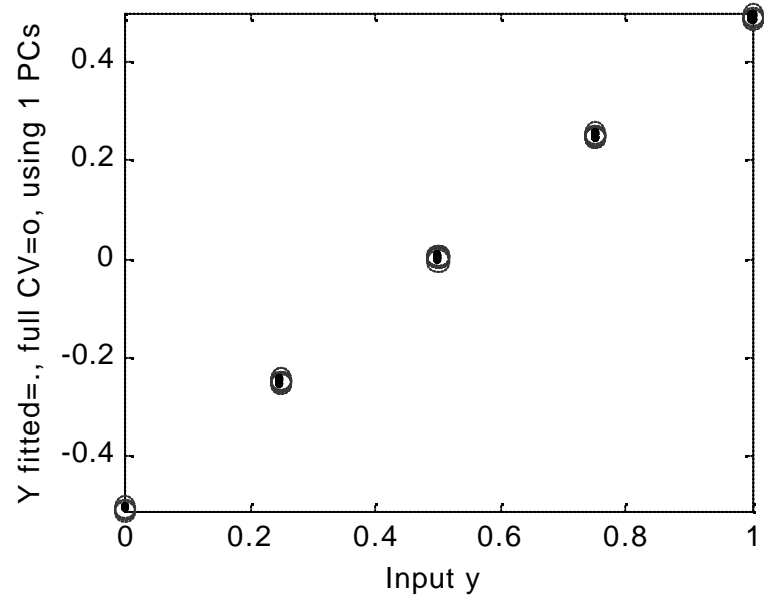
after pre-treatment



Cal. for y from input Z,  $r_{CV}=0.046$



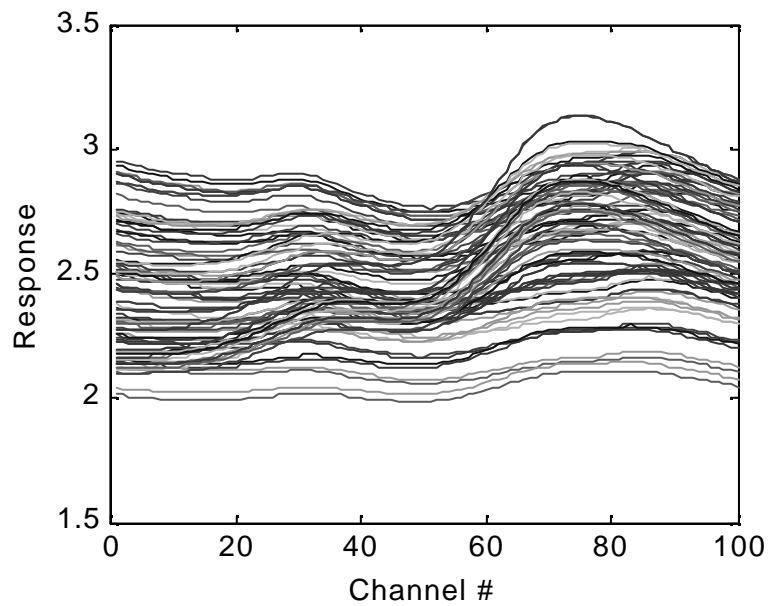
Cal. for y after EMSC/EISC,  $r_{CV}=1$



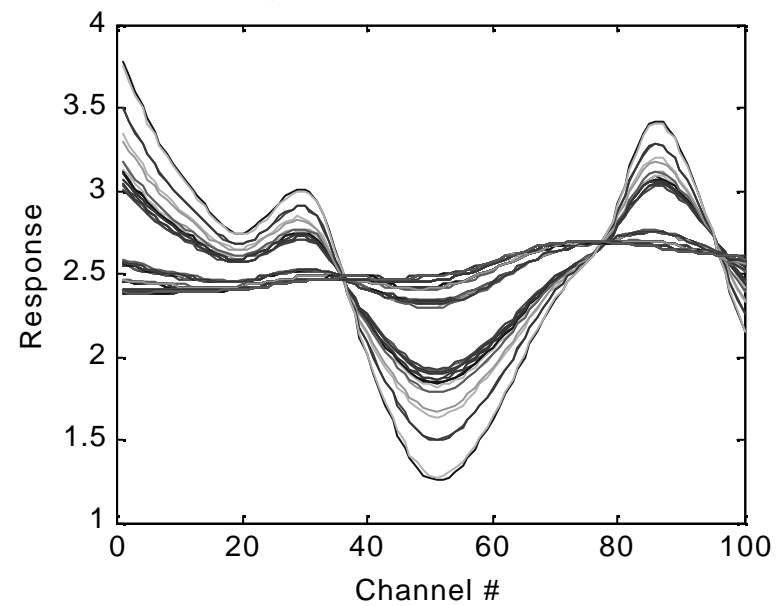


Comparisons of these models:

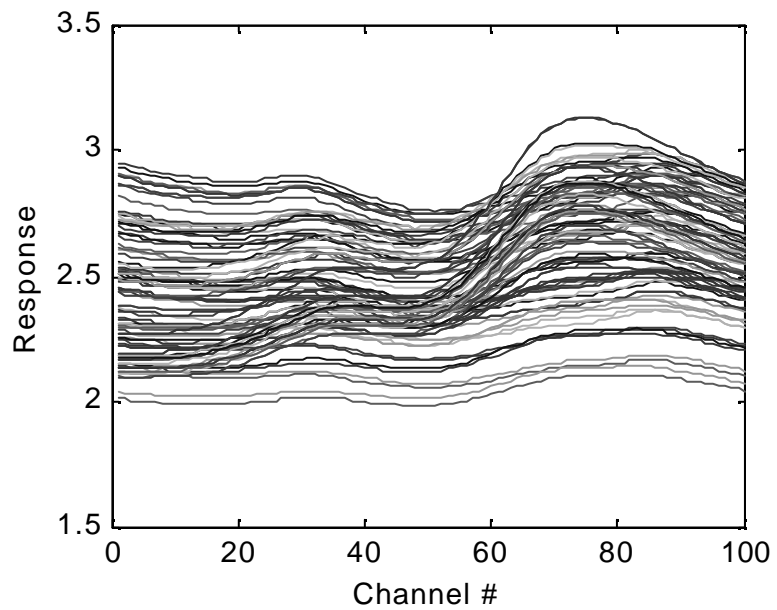
Input, EMSC<sub>z</sub>.MAT



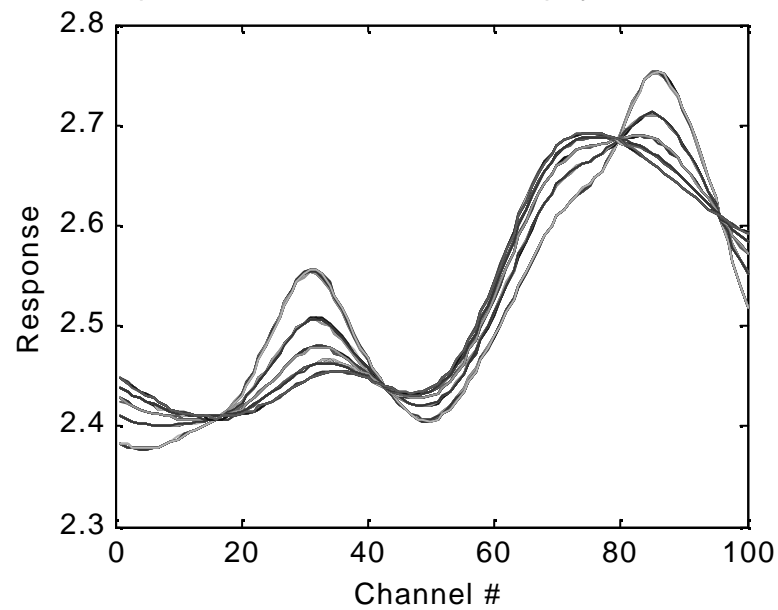
Output, DataCase=102, MSC



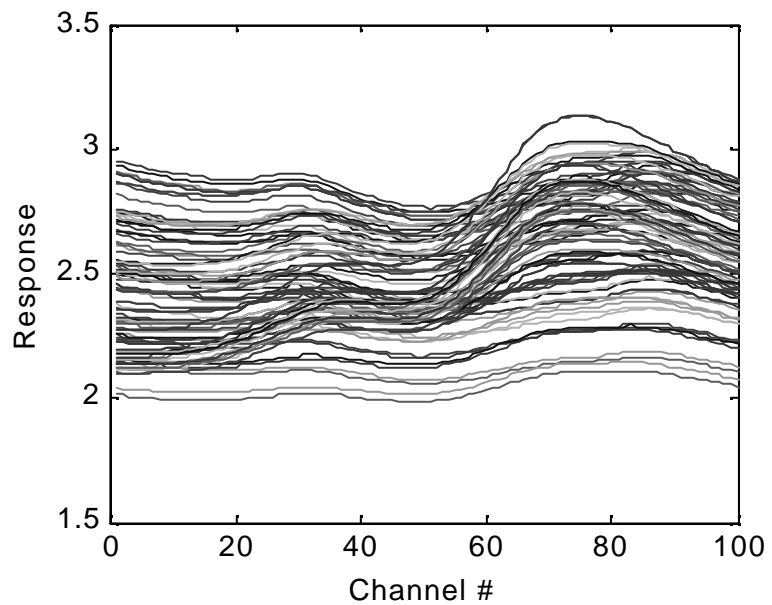
Input, EMSC<sub>z</sub>.MAT



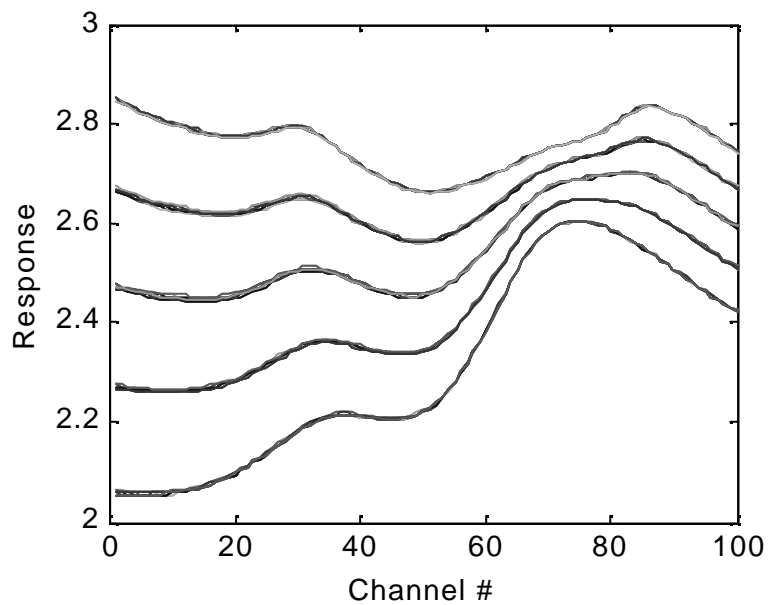
Output, DataCase=103, EMSC physical,default



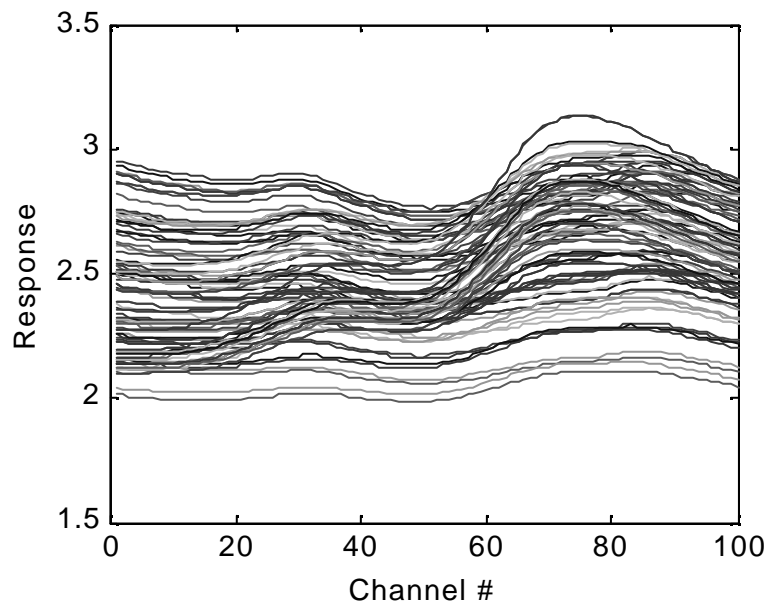
Input, EMSC<sub>z</sub>.MAT



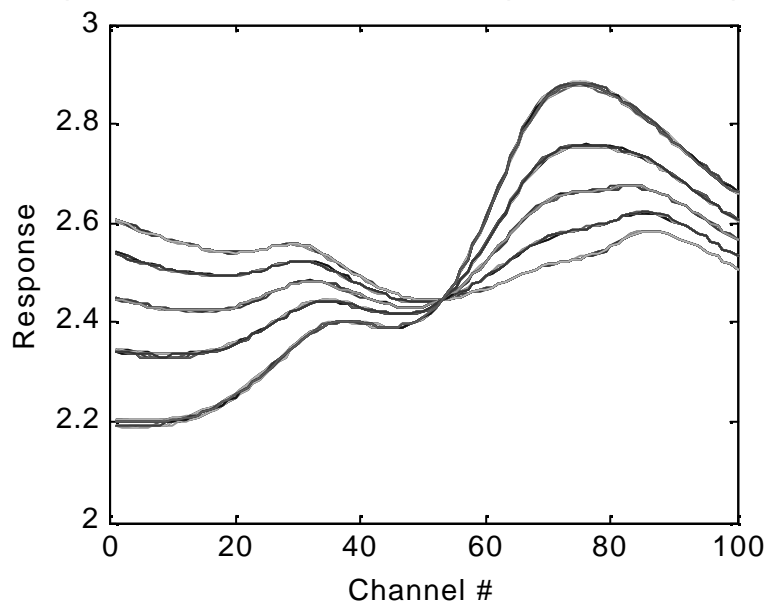
Output, DataCase=108, EMSC, physical & Good & Bad Spectra from



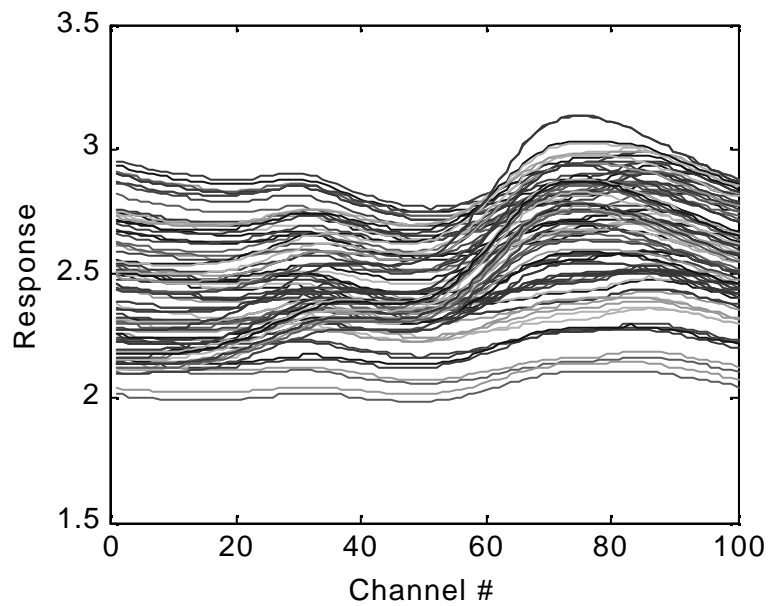
Input, EMSC<sub>z</sub>.MAT



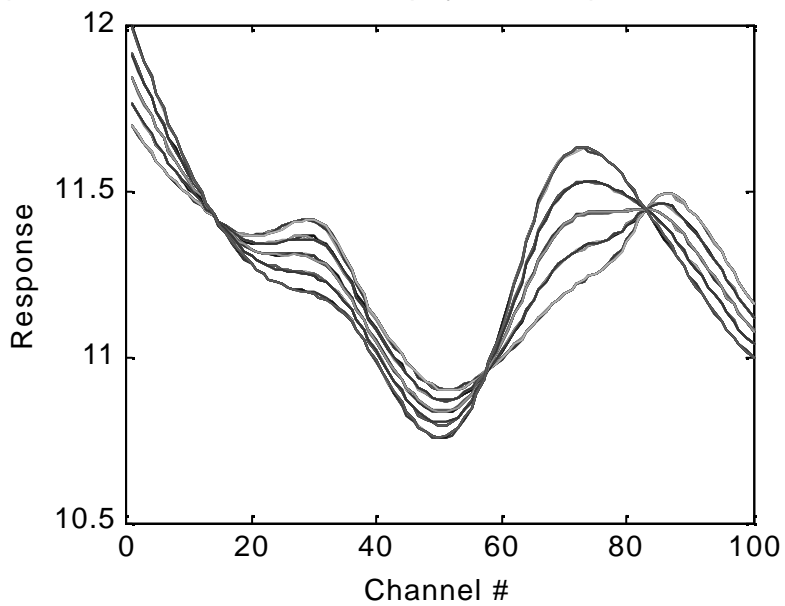
Output, DataCase=153, EMSC, opt. a Good component



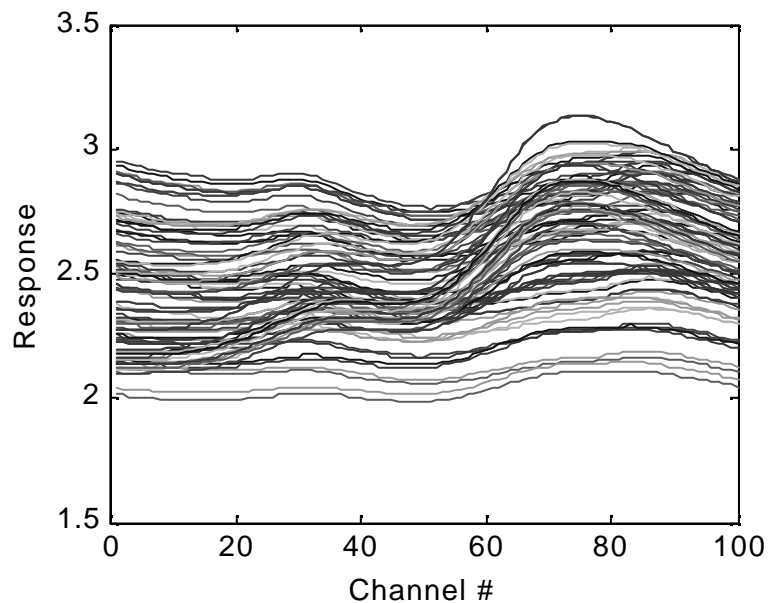
Input, EMSC<sub>z</sub>.MAT



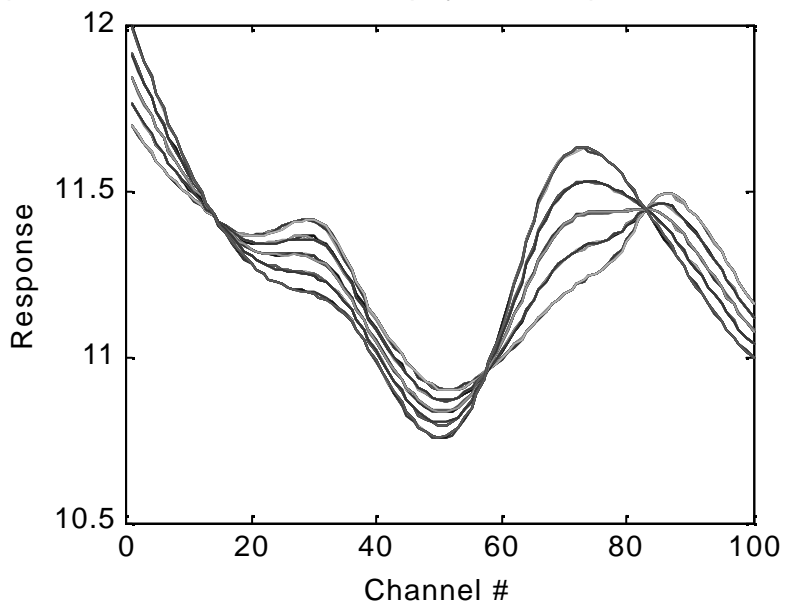
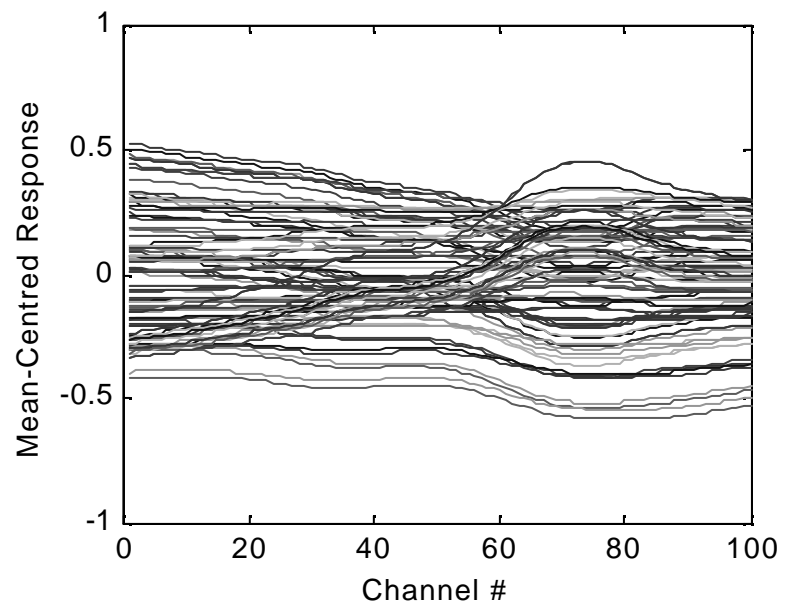
Output, DataCase=105, EMSC physical & opt. mean Ref spectr



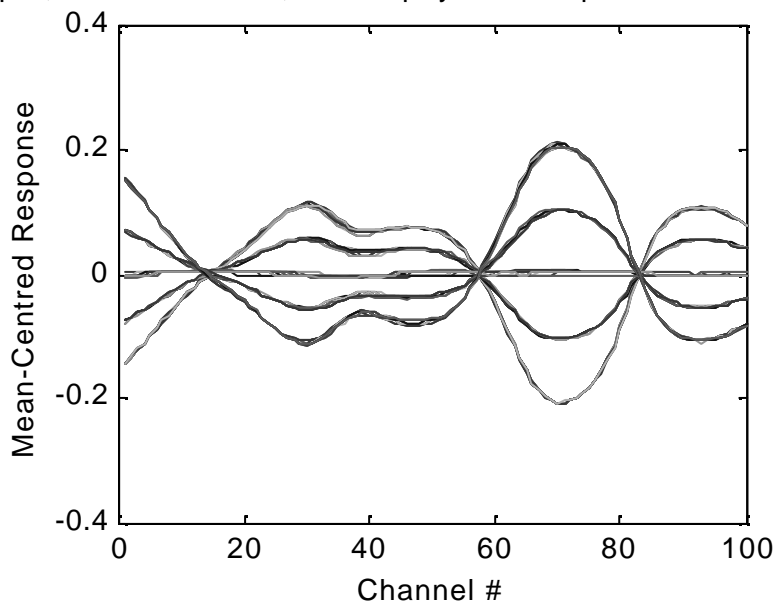


Input, EMSC<sub>Z</sub>.MAT

Output, DataCase=105, EMSC physical &amp; opt. mean Ref spectr

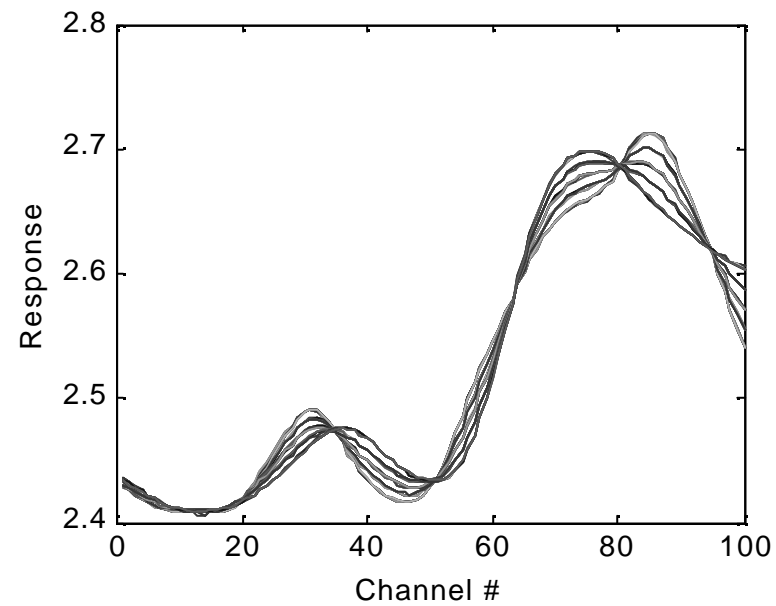
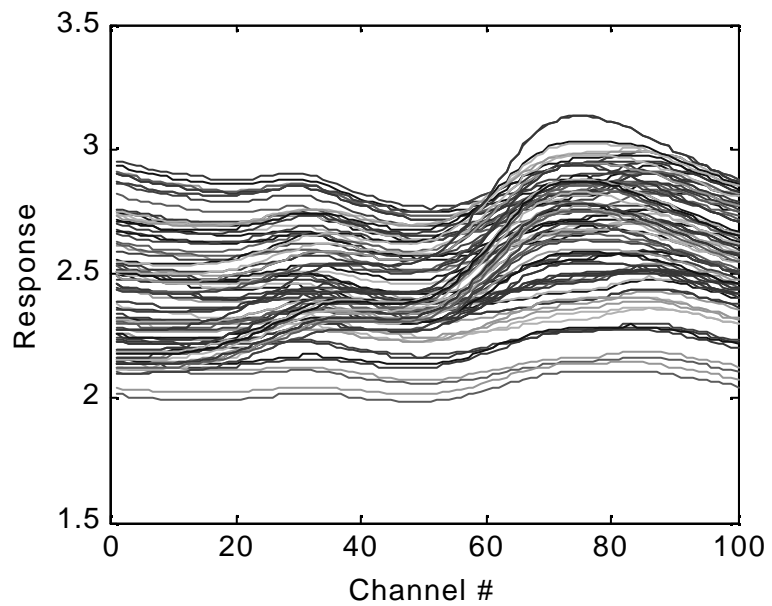
Input, EMSC<sub>Z</sub>.MAT

Output, DataCase=105, EMSC physical &amp; opt. mean Ref spectr



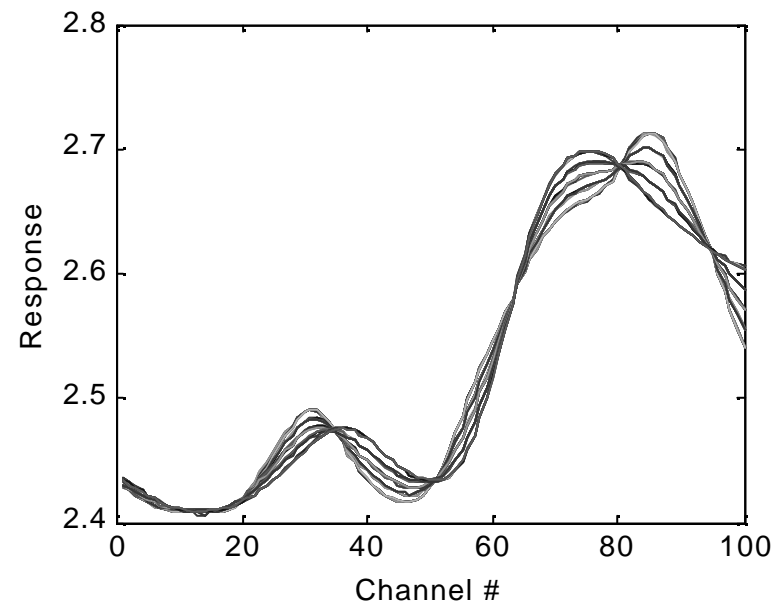
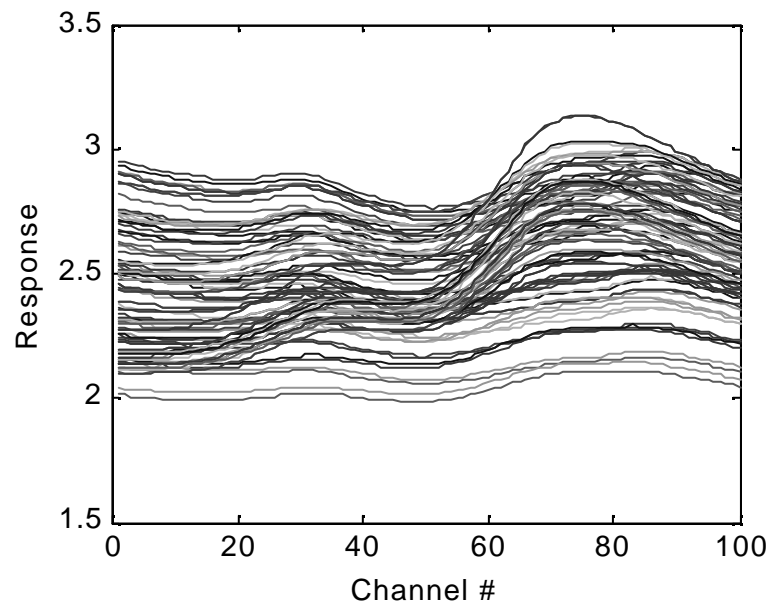
Input, EMSC<sub>z</sub>.MAT

Output, DataCase=155, EMSC, opt.an extra Bad spectrum, in addition to inp



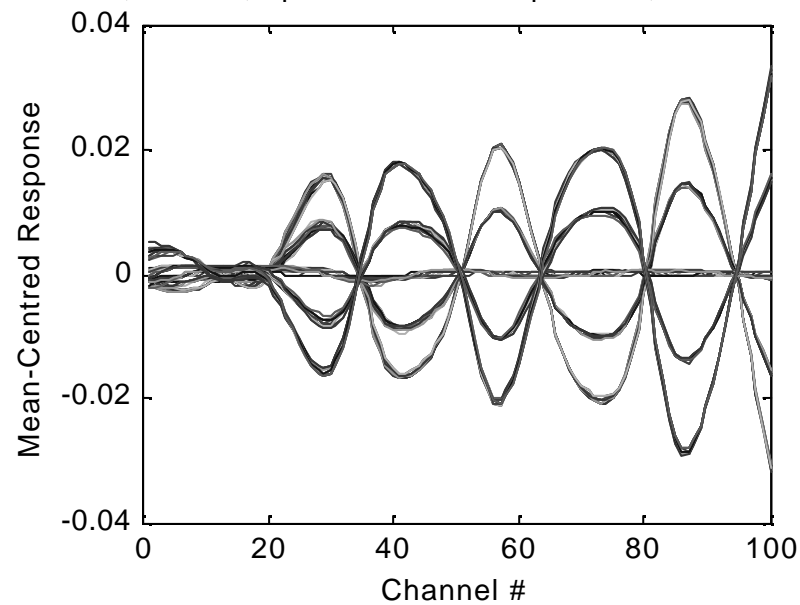
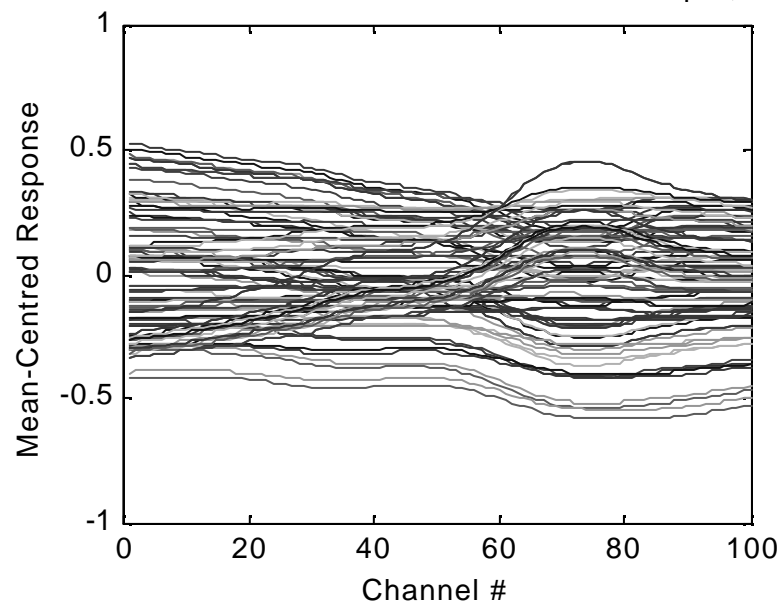
Input, EMSC<sub>z</sub>.MAT

Output, DataCase=155, EMSC, opt.an extra Bad spectrum, in addition to inp

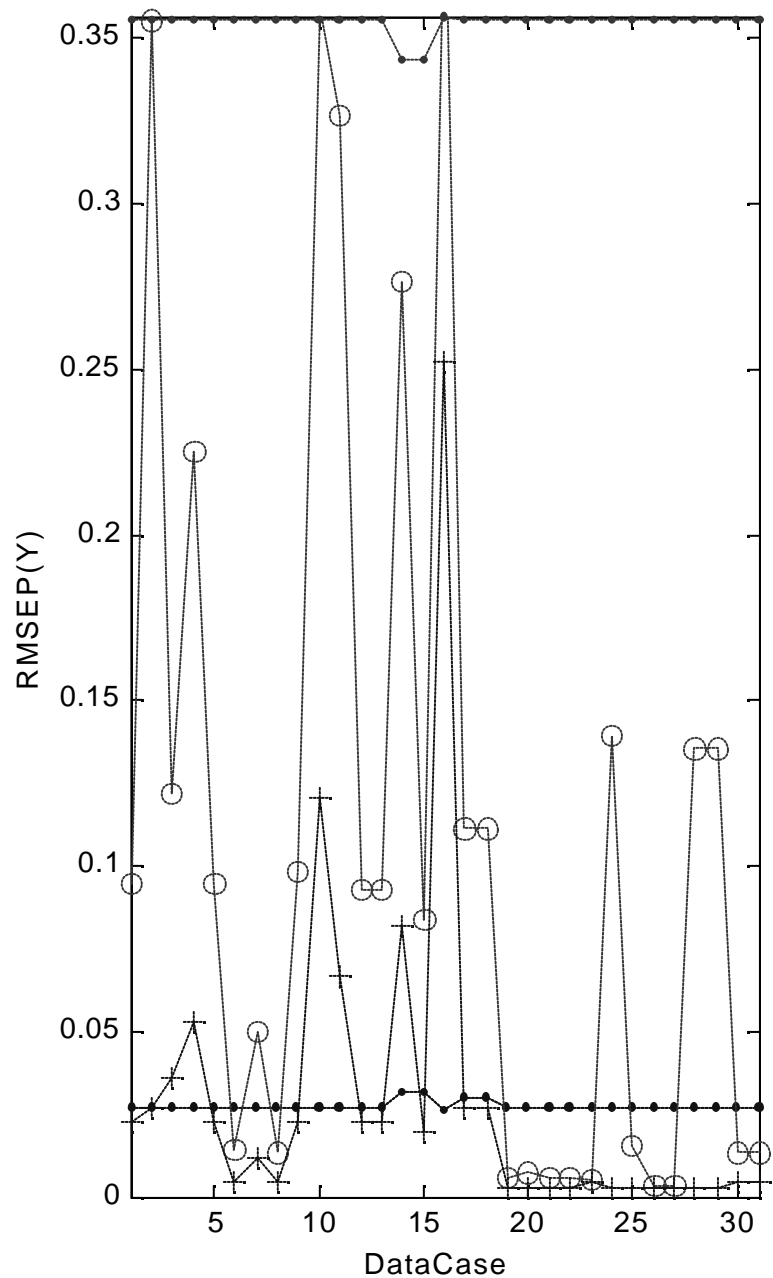


Input, EMSC<sub>z</sub>.MAT

Output, DataCase=155, EMSC, opt.an extra Bad spectrum, in addition to inp



Comparison of DataCases,  $r=1$  PC,  $b=A_{Opt}$  PCs; ...=input data



Automatic comparison of many different pre-processing alternatives for a given data set

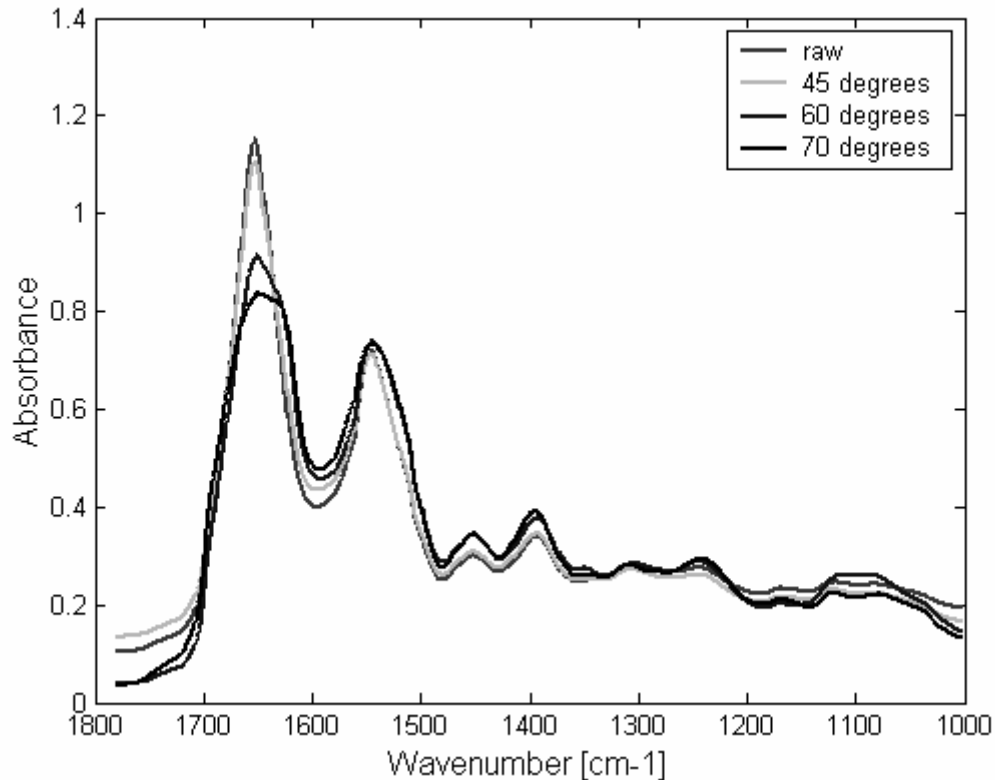


# Comparative study of chemometric and conventional pre-processing methods for

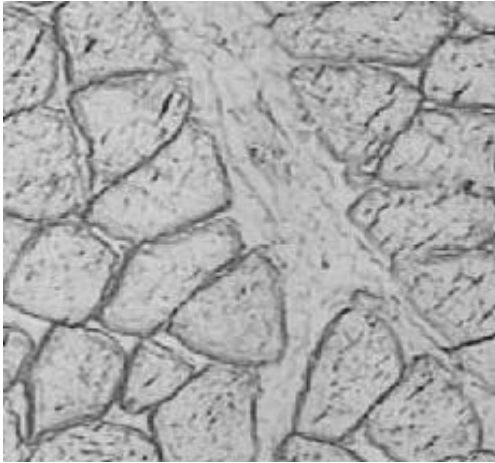
## FT-IR spectra of biological material

A. Kohler, C. Kirschner, A. Oust and H. Martens

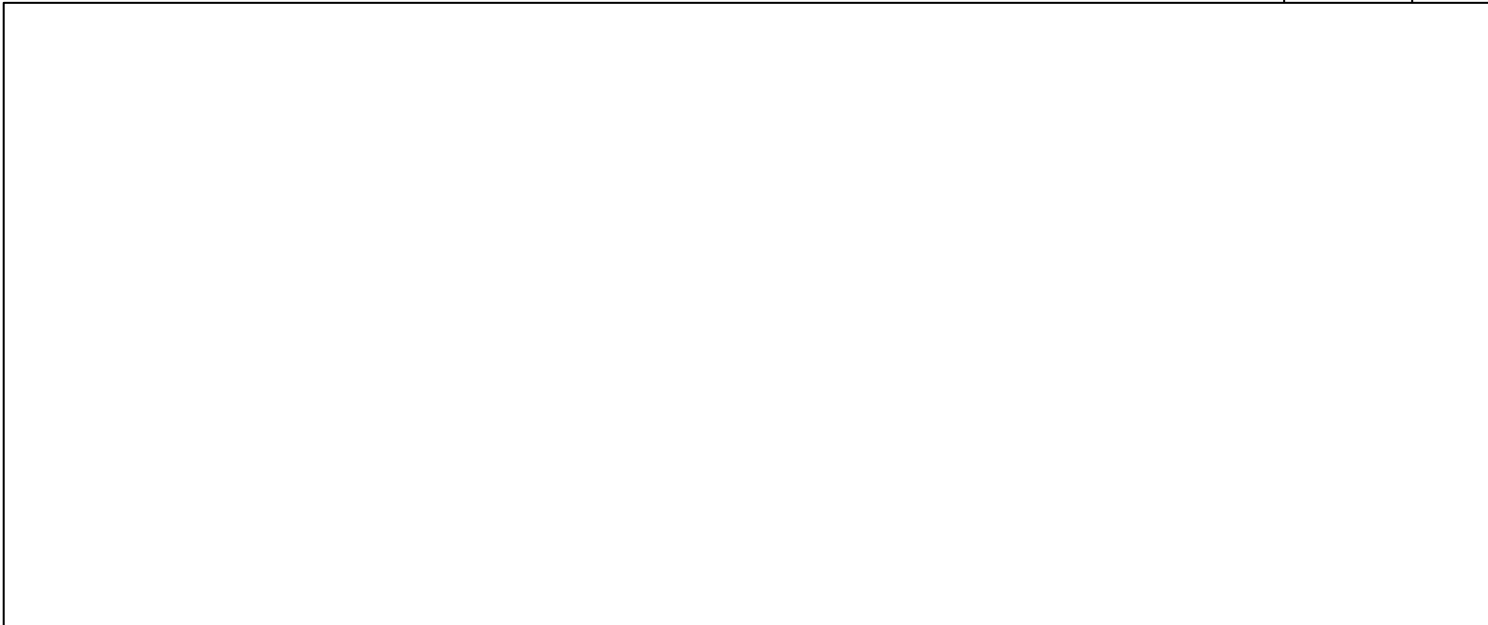
### FT-IR of heated bovine muscle:



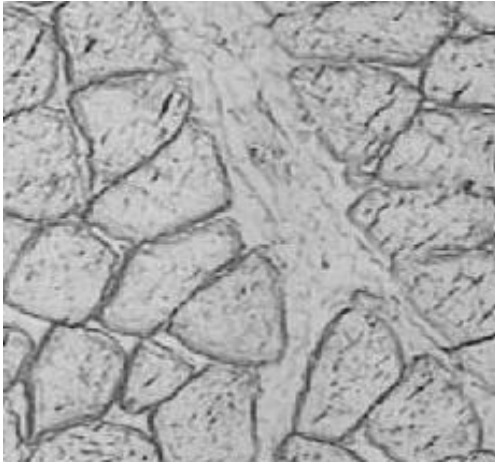
**Vis. image of muscle**



**Chemical map from  $I_{1630}/I_{1654}$   
as a measure for the denaturation level**



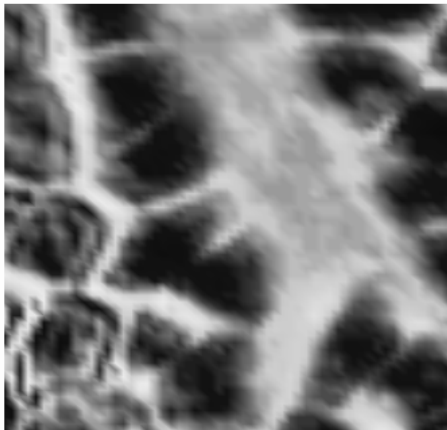
Vis. image of muscle



Chemical map from  $I_{1630}/I_{1654}$   
as a measure for the denaturation level



EMSC  $a_i$



EMSC  $b_i$





The correlation coefficients for

**X**= pre-processed FT-IR spectra of myofibres and

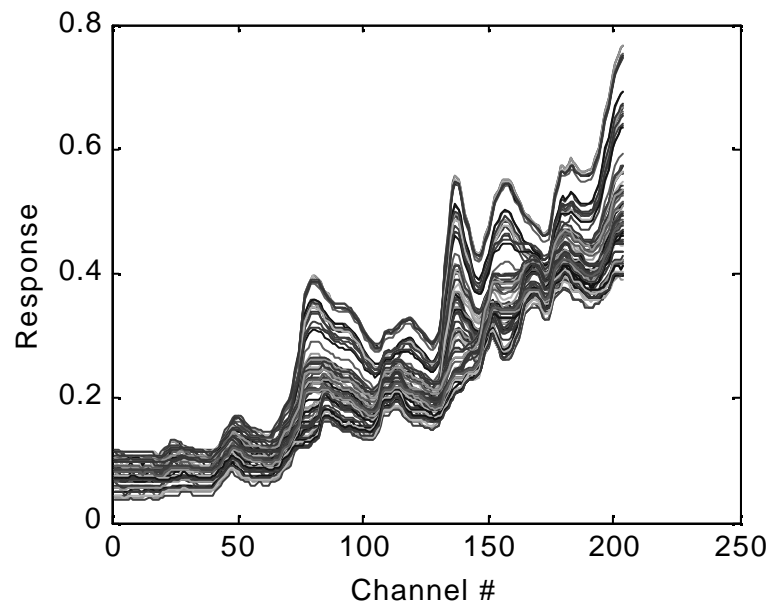
**y** = the different temperatures used for the heat treatment

1000-1780 cm <sup>-1</sup>	Original spectra	Derivative + vector normalisation	MSC	EMSC
Correlation coefficient All variables	0.92 (10 PCs)	0.91 (5 PCs)	0.92 (6 PCs)	0.94 (7 PCs)
Correlation coefficient Selected variables	0.93 (8 PCs, 120 variables)	0.91 (1 PC, 103 variables)	0.92 (2 PCs, 215 variables)	0.95 (3 PCs, 182 variables)

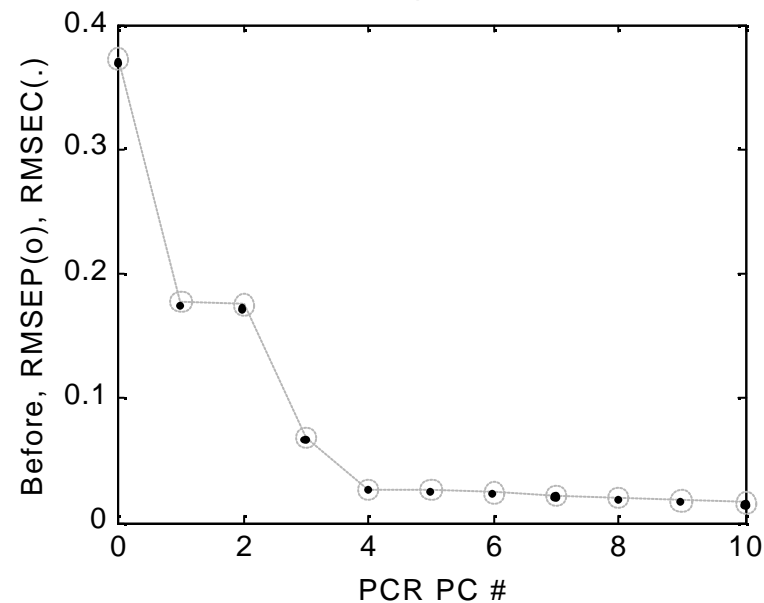
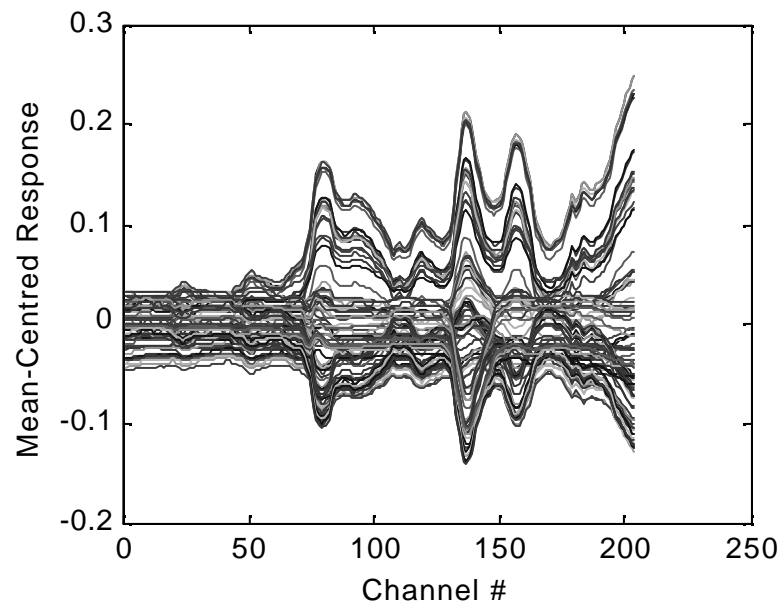
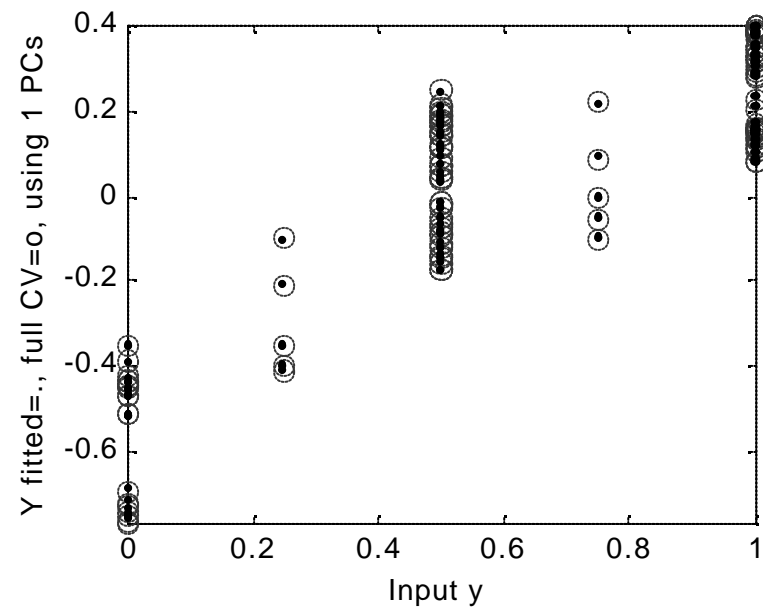


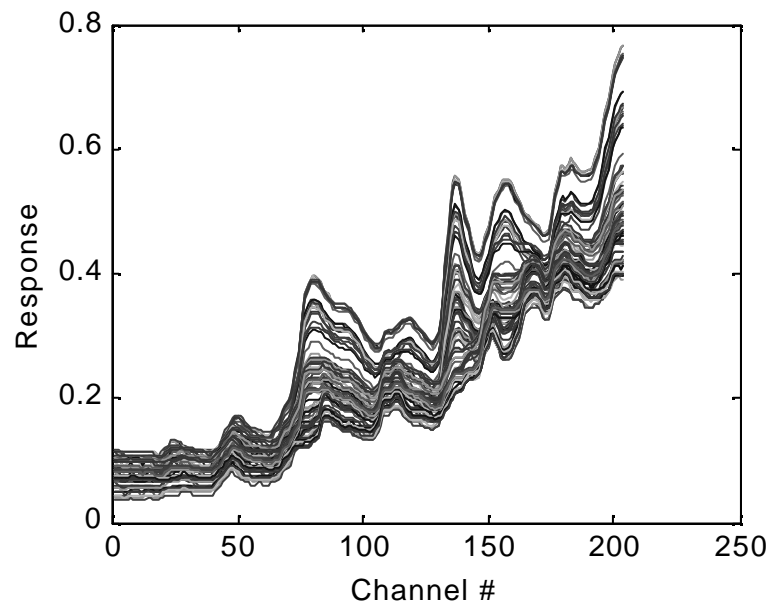
NIR reflectance of  
protein and starch mixtures (5 mixtures)  
in different bottles and  
at different water contents.

Data from Xuxin Lai, Biocentrum DTU

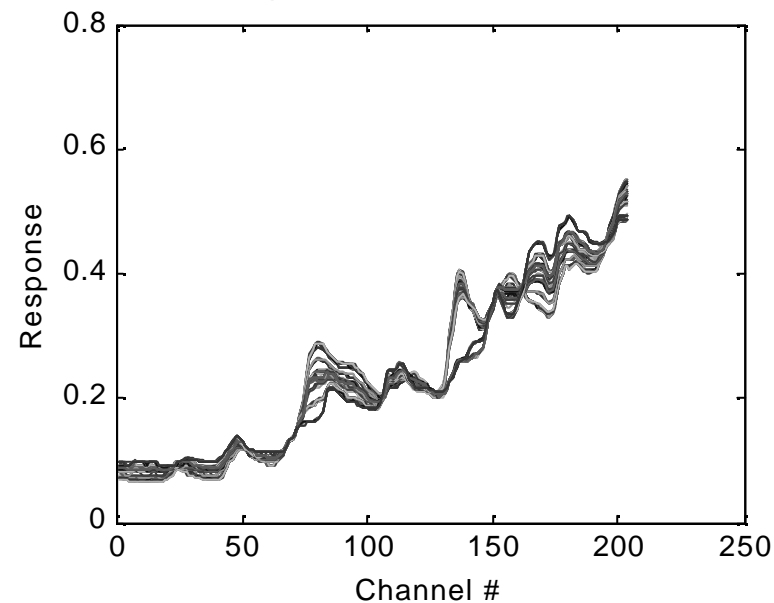
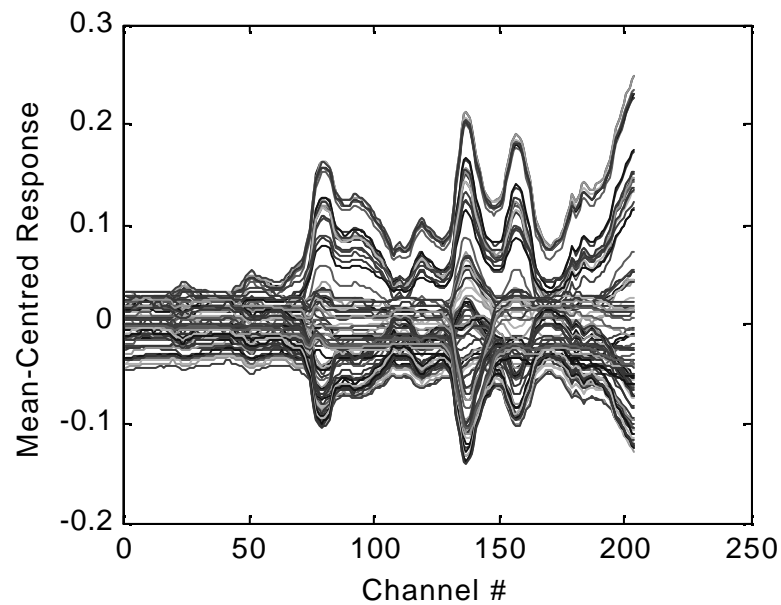
Input, EMSC<sub>Z</sub>.MAT

DataCase=100 No pre-treatment, before

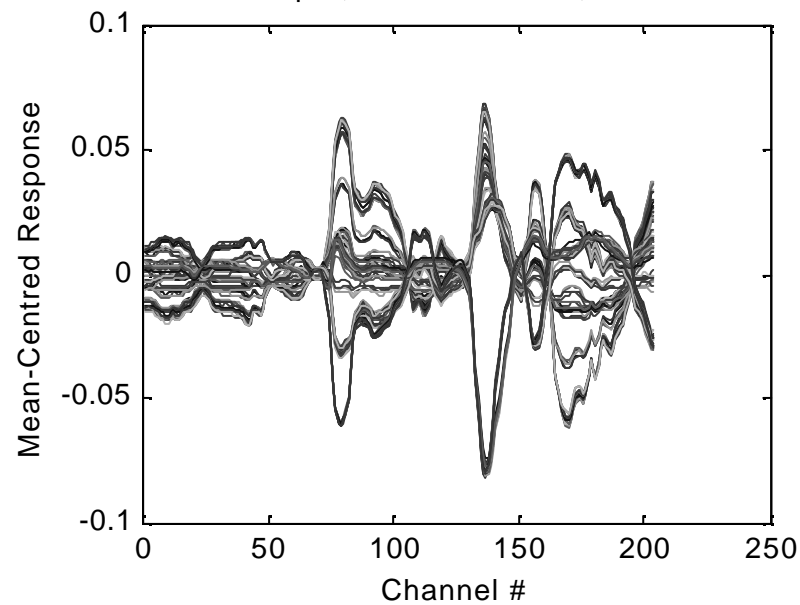
Input, EMSC<sub>Z</sub>.MATCal. for y from input Z,  $r_{CV}=0.879$ 

Input, EMSC<sub>z</sub>.MAT

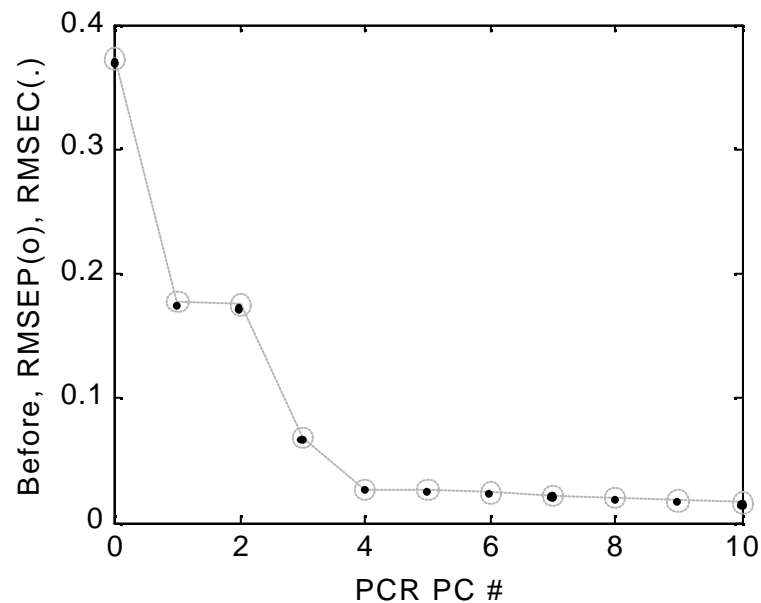
Output, DataCase=102, MSC

Input, EMSC<sub>z</sub>.MAT

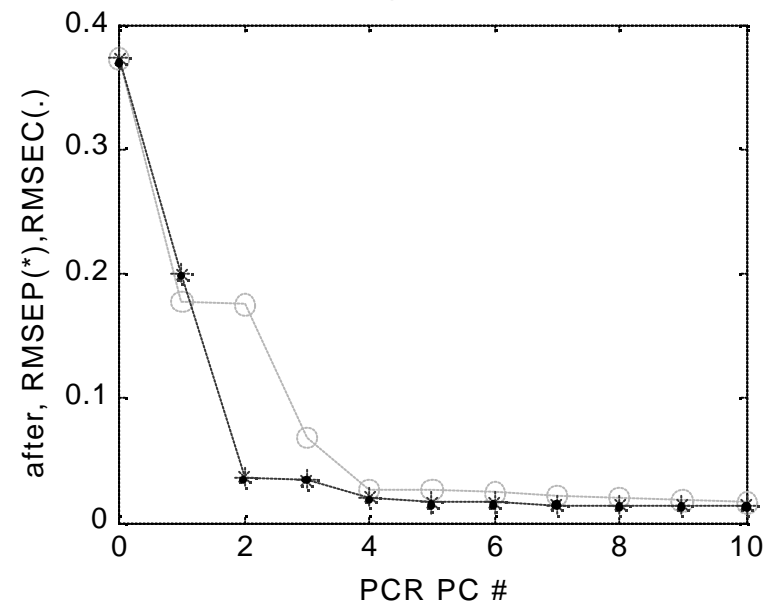
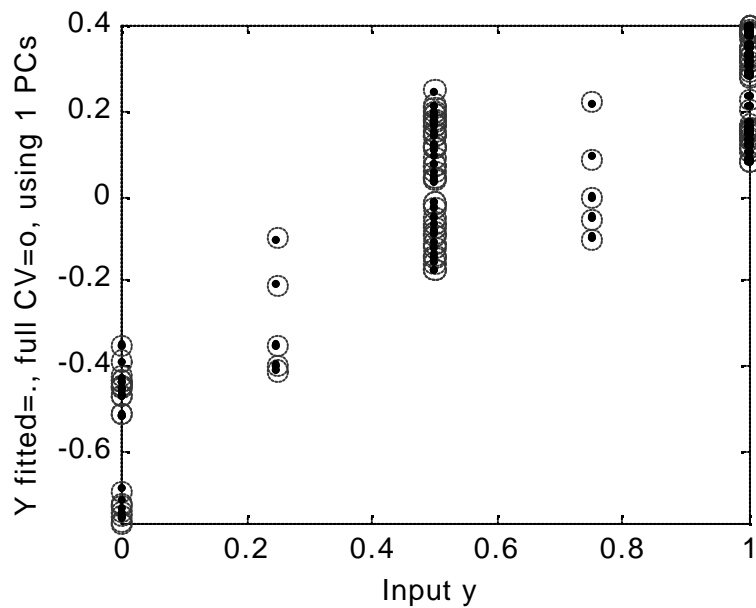
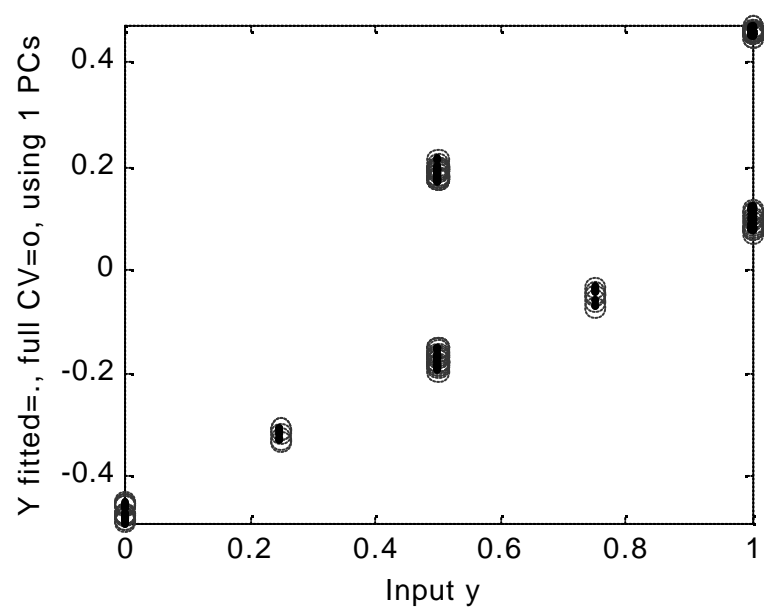
Output, DataCase=102, MSC



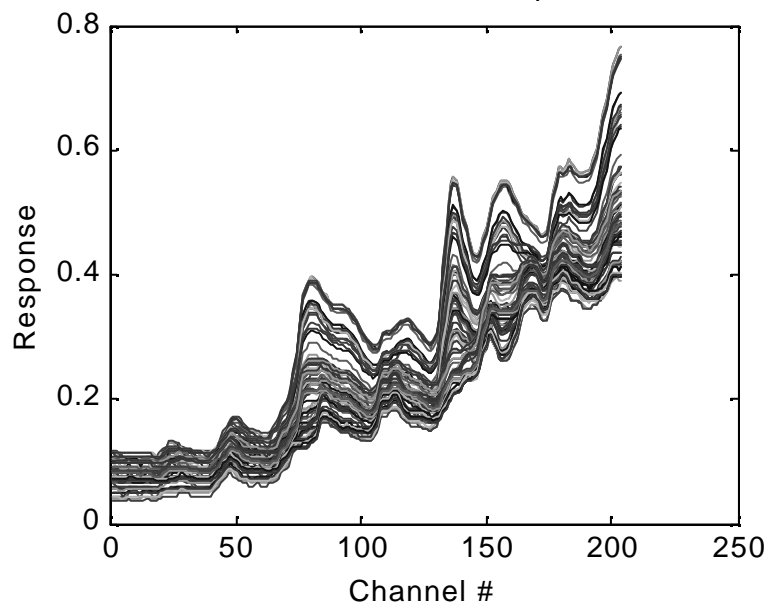
DataCase=102 MSC, before



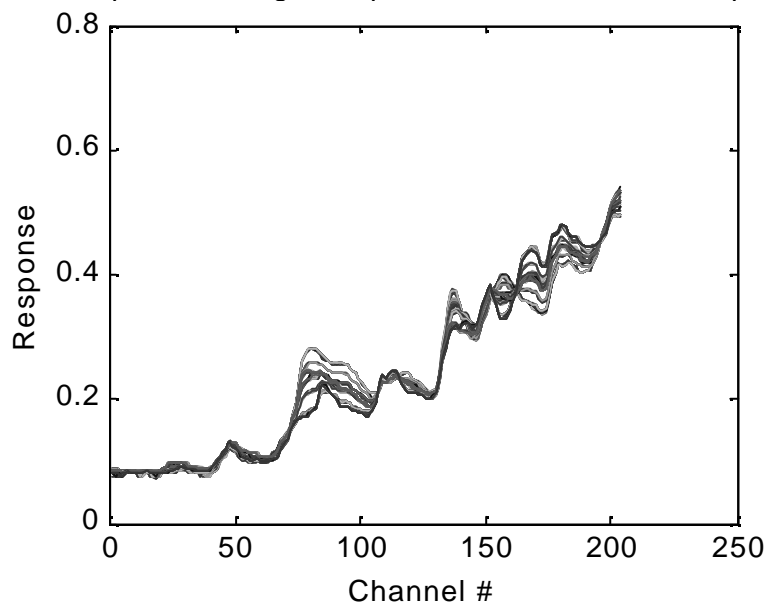
after pre-treatment

Cal. for y from input Z,  $r_{CV}=0.879$ Cal. for y after EMSC/EISC,  $r_{CV}=0.843$ 

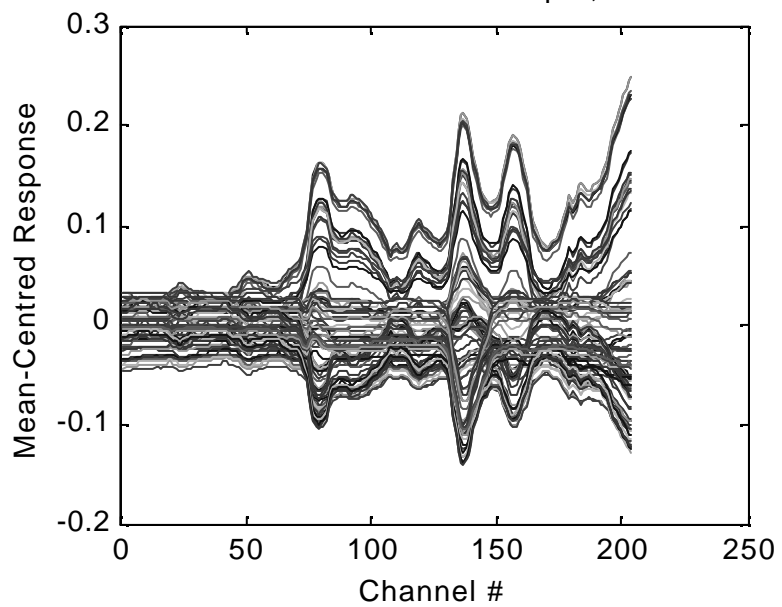
Input, EMSC\_MAT



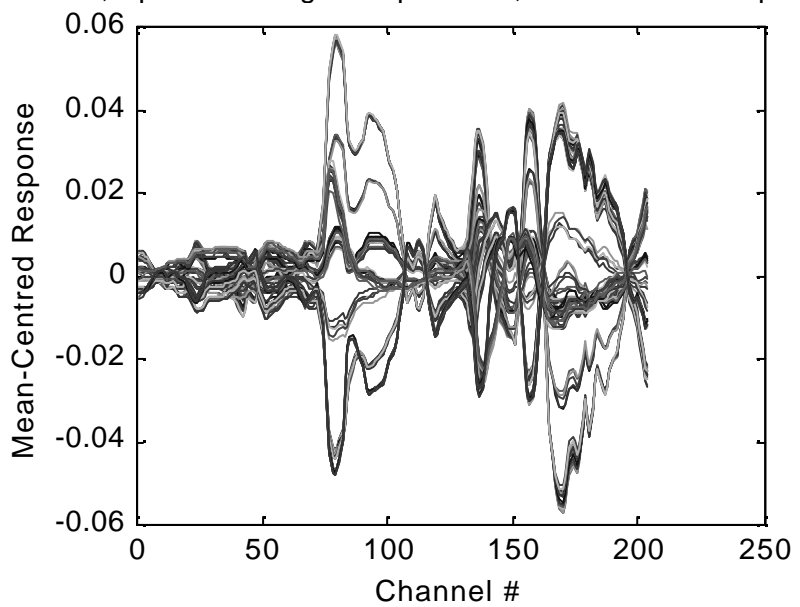
Output, DataCase=161, EMSC, opt. 3D one good spectrum, in addition to input Good



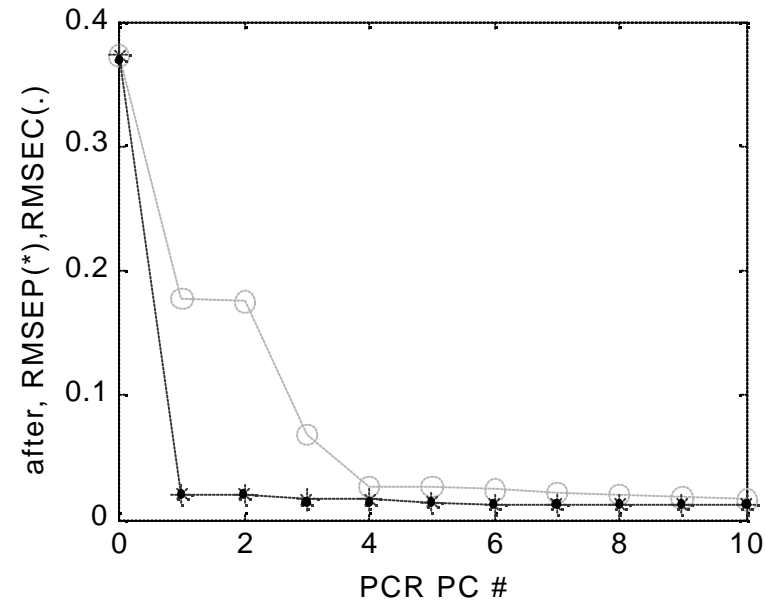
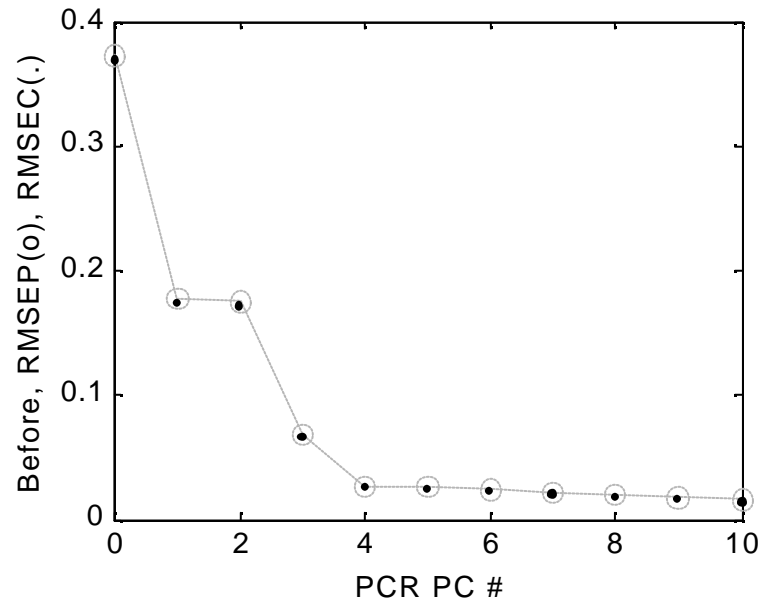
Input, EMSC\_MAT



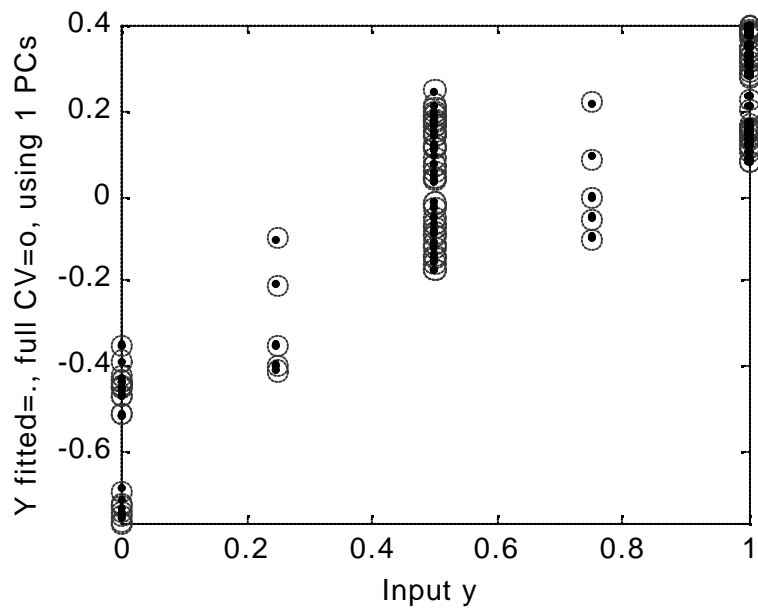
Output, DataCase=161, EMSC, opt. 3D one good spectrum, in addition to input Good



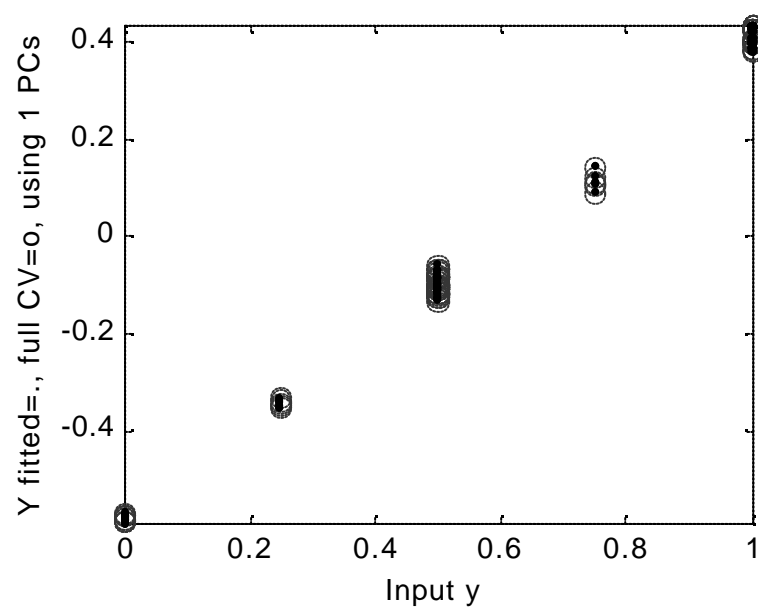
SC, opt. 3D one good spectrum, in addition to input GoodSpectra and BadSpectra, before pre-treatment



Cal. for y from input Z,  $r_{CV}=0.879$



Cal. for y after EMSC/EISC,  $r_{CV}=0.999$







# NIT whole wheat grains

Pedersen, D.K., Martens, H., Pram Nielsen, J. and Balling Engelsen, S. Light absorbance and light scattering separated by Extended Inverted Multiplicative Signal Correction (EIMSC). Analysis of NIT spectra of single wheat seeds. Applied Spectroscopy 2002, **56**(9) 1206-1214.

# EISC vs EMSC

**MSC:**  $\mathbf{z}_i = a_i \mathbf{1}' + b_i \mathbf{m}' + \mathbf{e}_i$

**EMSC:**  $\mathbf{z}_i = a_i \mathbf{1}' + b_i \mathbf{m}' + h_i \mathbf{k}' + d_i \mathbf{l} + e_i \mathbf{l}^2 + \mathbf{e}_i$

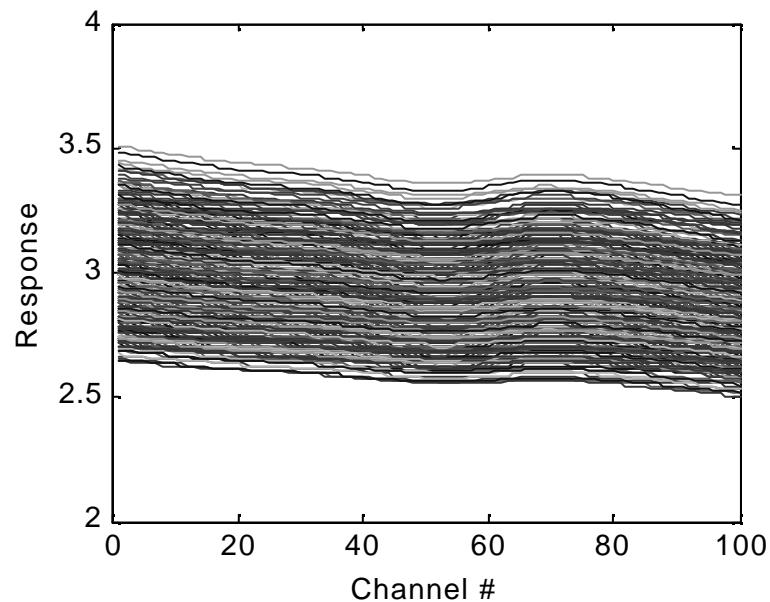
$$\mathbf{z}_{i,\text{Corrected}} = (\mathbf{z}_i - a_i \mathbf{1}' - d_i \mathbf{l} - e_i \mathbf{l}^2) / b_i$$

**ISC:**  $\mathbf{m}' = a_i \mathbf{1}' + b_i \mathbf{z}_i + \mathbf{g}_i$

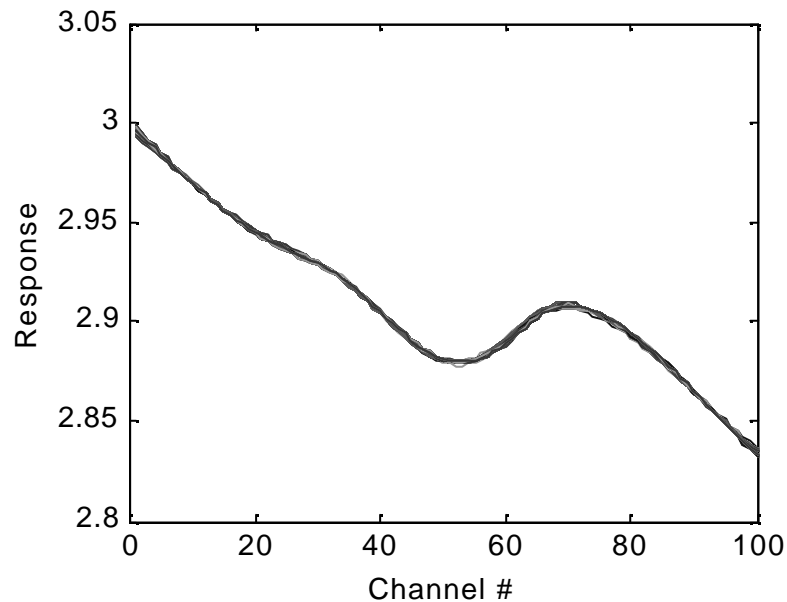
**EISC:**  $\mathbf{m}' = a_i \mathbf{1}' + b_i \mathbf{z}_i + h_i \mathbf{k} + d_i \mathbf{l} + e_i \mathbf{l}^2 + \mathbf{g}_i$

$$\mathbf{z}_{i,\text{Corrected}} = a_i \mathbf{1}' + b_i \mathbf{z}_i + h_i \mathbf{k} + d_i \mathbf{l} + e_i \mathbf{l}^2$$

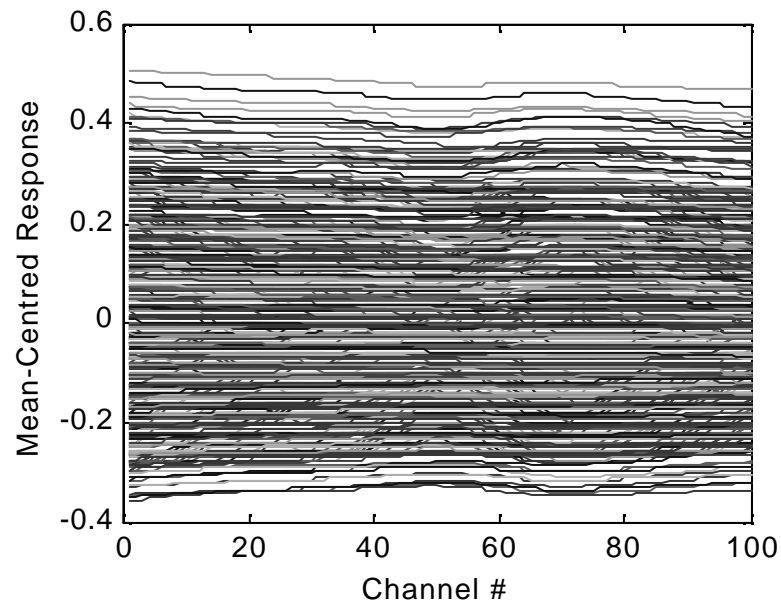
Input, EMSC<sub>z</sub>.MAT



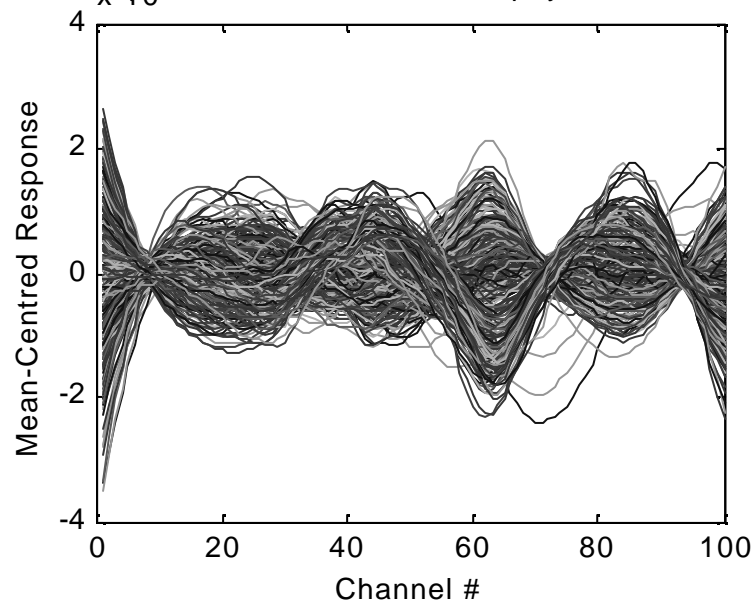
Output, DataCase=202, EISC physical,default



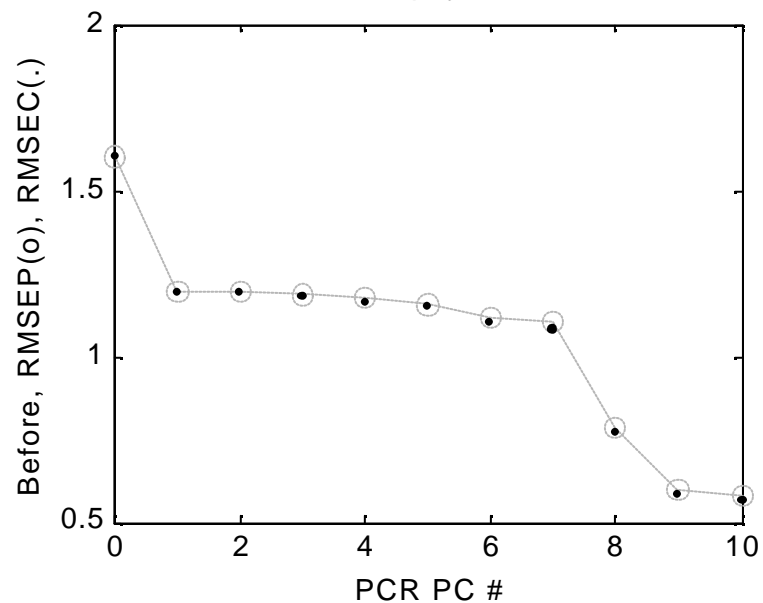
Input, EMSC<sub>z</sub>.MAT



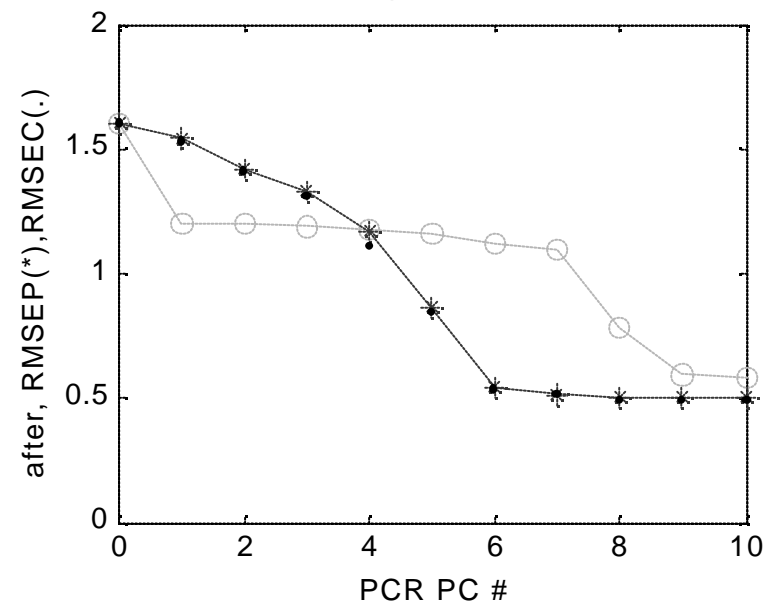
Output, DataCase=202, EISC physical,default



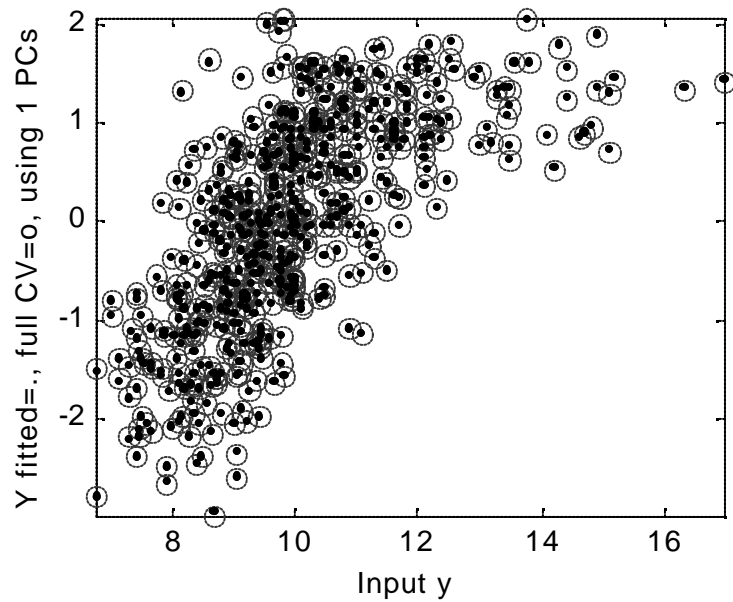
DataCase=202 EISC physical,default, before



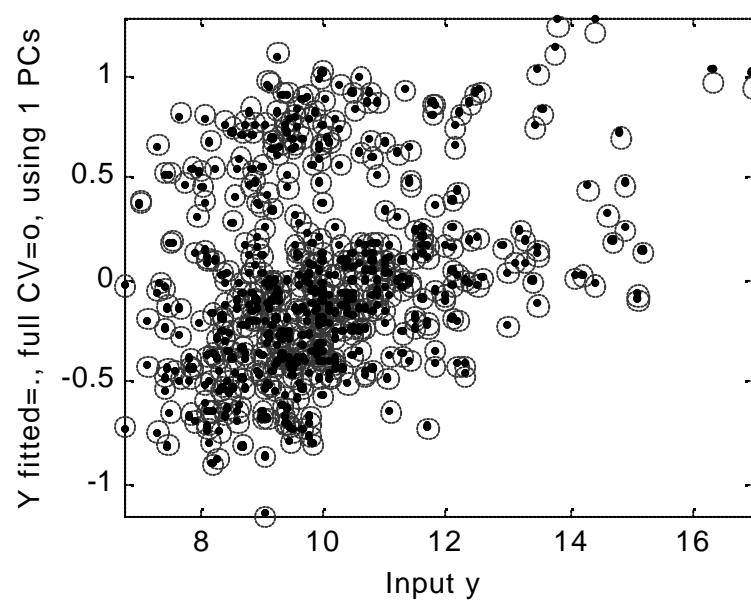
after pre-treatment



Cal. for y from input Z,  $r_{CV}=0.666$



Cal. for y after EMSC/EISC,  $r_{CV}=0.274$



New method  $\approx$  Direct Orthogonalization:

Estimate unknown “good” and “bad” spectra after projection of spectra  $\mathbf{Z}$  on  $\mathbf{y}$ :

$$\mathbf{Z} = \mathbf{y}\mathbf{k}_{\text{Good}}' + \mathbf{E}$$

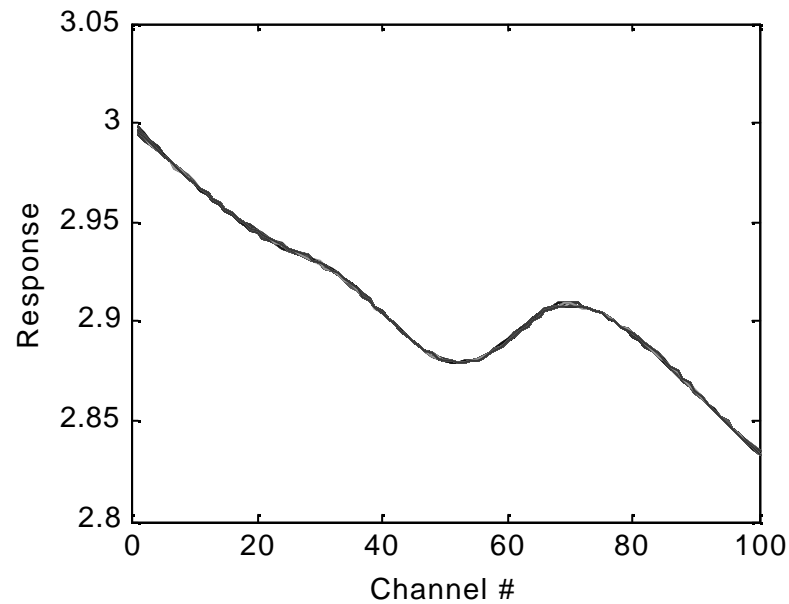
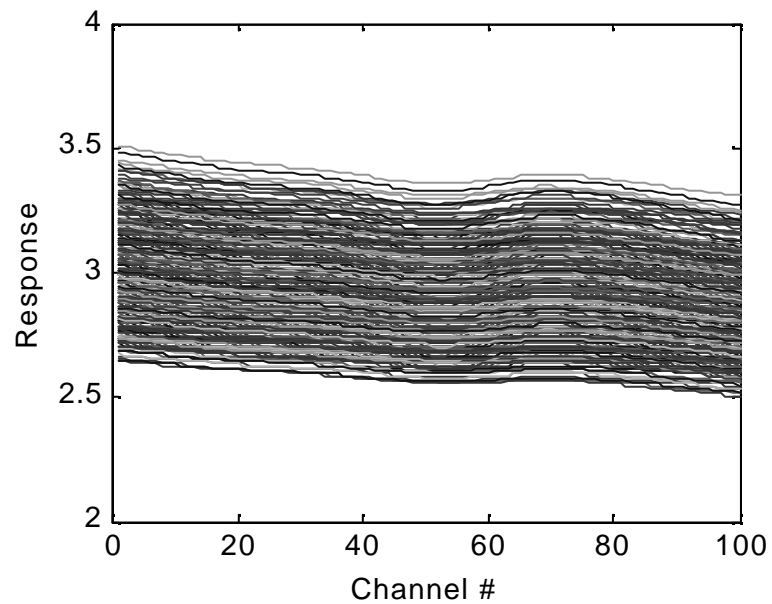
Estimate  $\mathbf{k}_{\text{Good}}$  and  $\mathbf{E}$  by regression

$$\mathbf{k}_{\text{Bad},1,2} = \text{svd}(\mathbf{E})$$

In EMSC: Estimate their concentrations, and subtract the effects of the “bad” spectra.

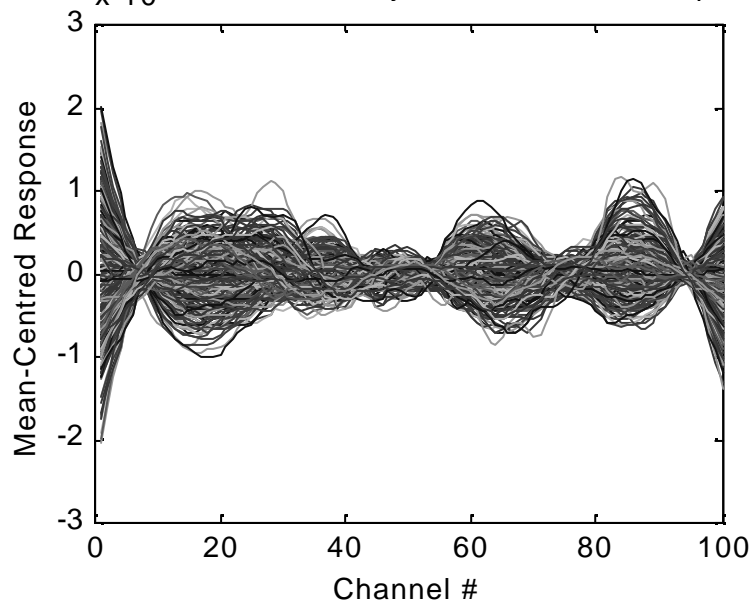
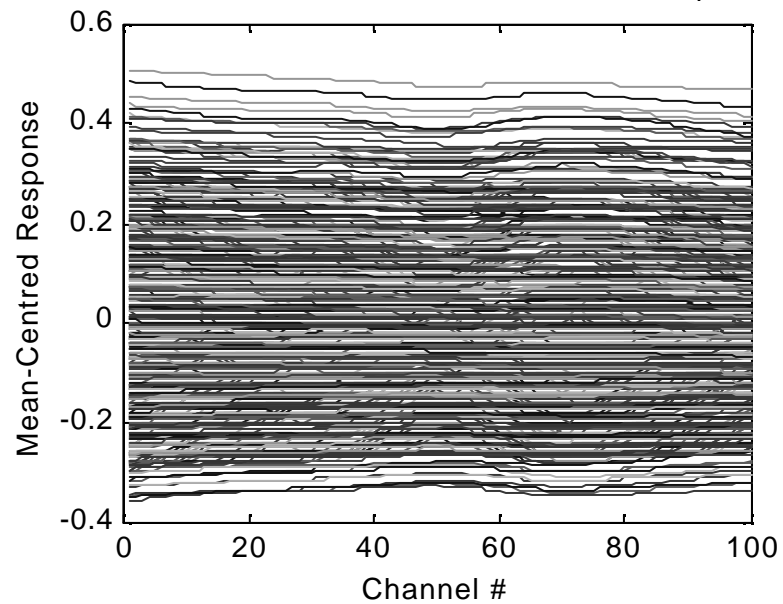
Input, EMSC<sub>z</sub>.MAT

Output, DataCase=122, EMSC, Automatically estimated 1 GoodSpectra and

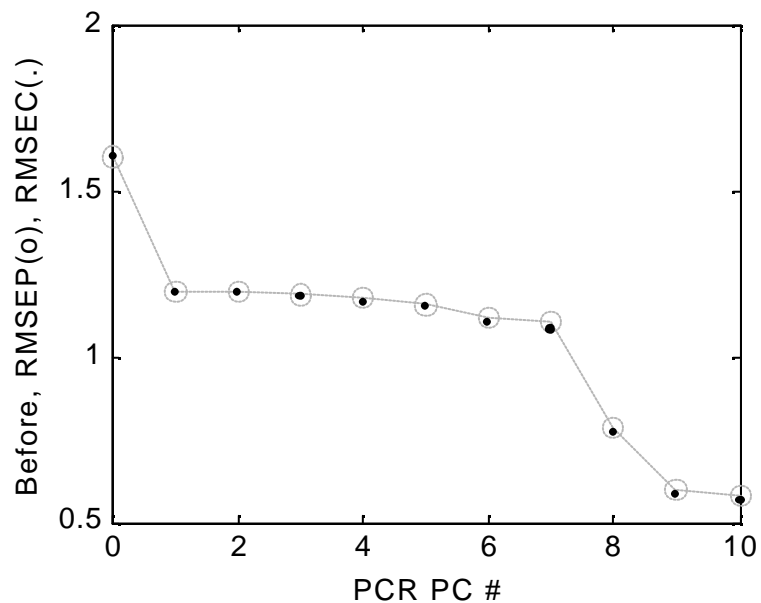


Input, EMSC<sub>z</sub>.MAT

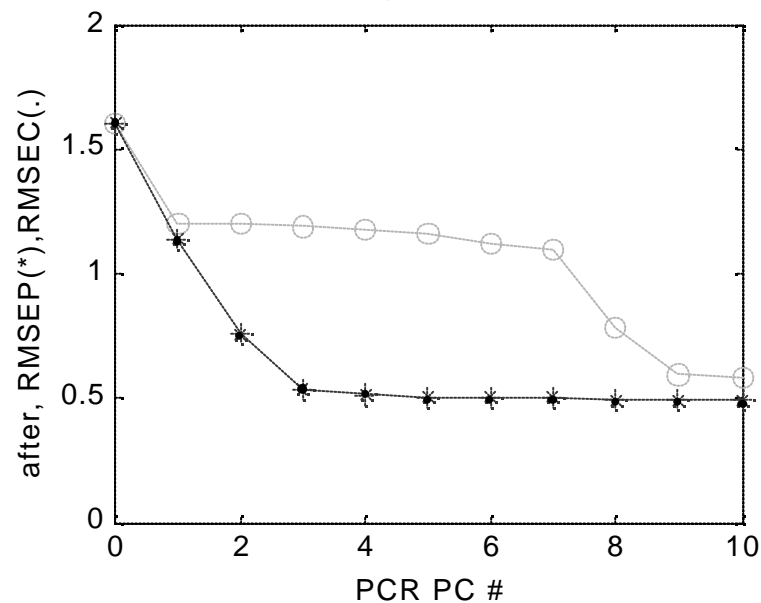
Output, DataCase=122, EMSC, Automatically estimated 1 GoodSpectra and



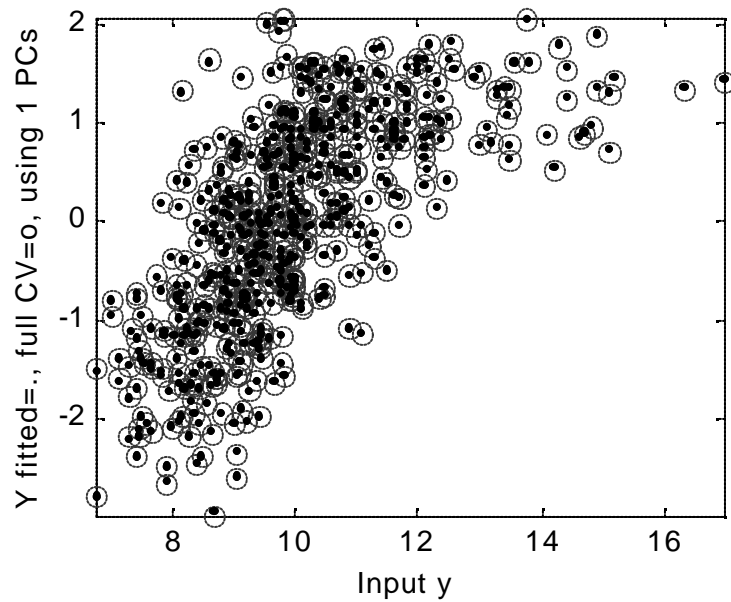
n=122 EMSC, Automatically estimated 1 GoodSpectra and 2 BadSpectra, before



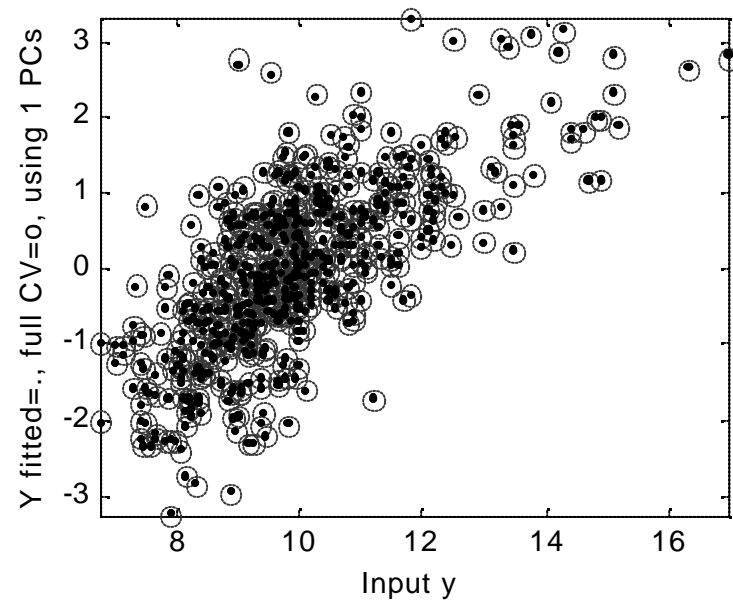
after pre-treatment



Cal. for y from input Z,  $r_{CV}=0.666$



Cal. for y after EMSC/EISC,  $r_{CV}=0.707$







# Summary

Have shown:

MSC  
    \     EMSC (physical effects)

# Summary

Have shown:

MSC

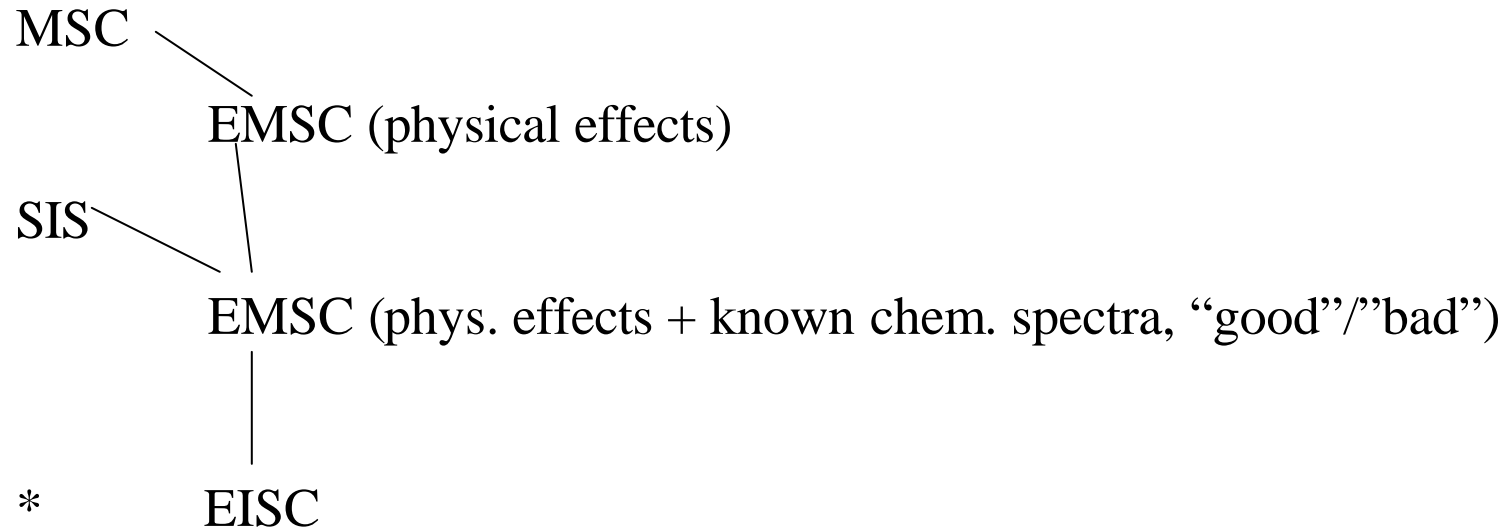
EMSC (physical effects)

SIS

EMSC (phys. effects + known chem. spectra, “good”/”bad”)

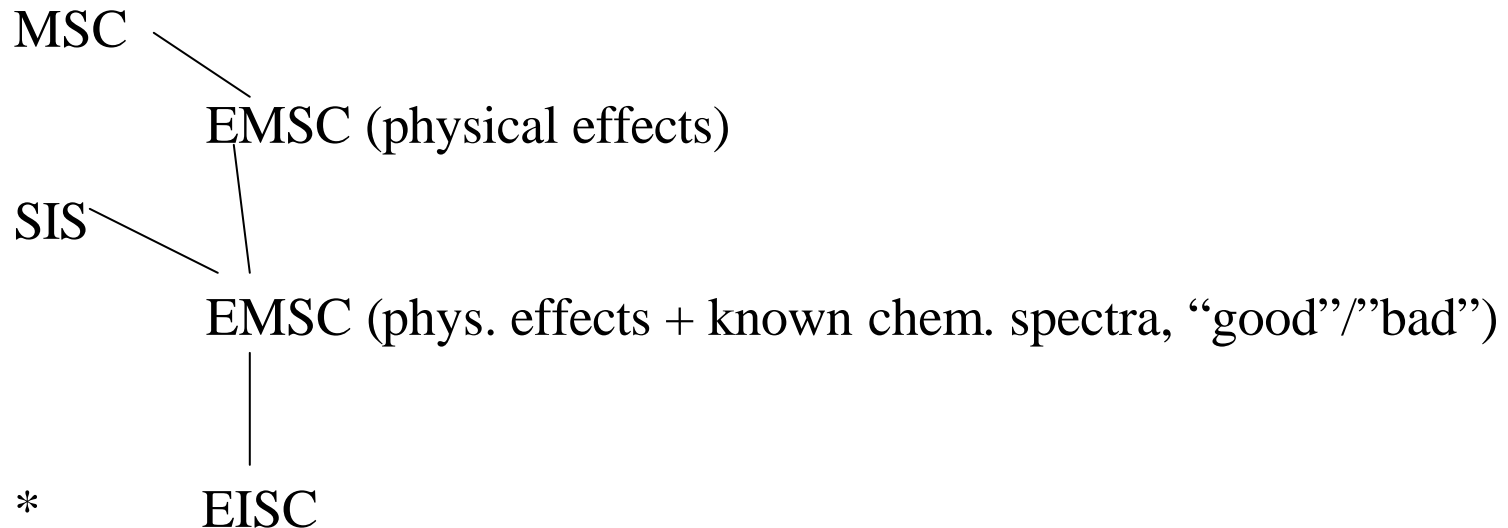
# Summary

Have shown:



# Summary

Have shown:



- \* Automatically *estimated* chem. spectra, (“good”/”bad”)
- \* Automatically *optimized* model spectra (ref, “good” or ”bad”) spectra
- \* *Re-estimated weights* for the LS parameter estimation in EMSC/EISC

## **Conclusions:**

- \* Separated chemical and physical information  
by model-based pre-processing.**
- \* Reduced required # of PCs in calibration model,  
from 5 to 1, which is expected chemically.**
- \* Many different appearances of the same input data set,  
depending on the preprocessing.  
Rotations in a subspace!**

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Rotations in a subspace!**

**<http://www.models.kvl.dk/source/EMSCtoolbox/index.asp>**