Godfrey's Island Rule
and the Indonesian Through Flow

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Abstract

The Island Rule describes the total transport around an island in an ocean basin. According to the rule, the transport is determined by the long-path integral of the surface wind stress around the island. An altered contour is used for the integration, in order to avoid dealing with westward intensification. The resulting transport yielded by the Island Rule is tested against simulation results of the Indonesian through flow (ITF) for different wind stresses in the Southern Ocean. The used simulation program Veros is provided by the Niels Bohr Institute. It is shown that the ITF transport increases the larger the wind stresses in the Southern Ocean. However, results yielded by the Island Rule show a more sensible behavior towards wind changes than simulation results. It is concluded that other driving factors of the ITF than surface wind stresses are to be considered. Meridional transport, nonlinearities and stratification are further suggested factors to be of importance for the estimation of the ITF transport.

Contents

1 Introduction

The behavior of the ocean on a global scale can be influenced by regional oceanic circulation. Because of this, regional circulations are to be considered, when analyzing global climate, as Earth's oceans and atmosphere are strongly coupled. In order to understand and predict global climate and its phenomenons, the research on one of these regional ocean sections, namely the Indonesian Passage, is of importance.

The Indonesian Through Flow (ITF) is an ocean current flowing through the Indonesian archipelago. The ITF enables warm water transport from the Pacific into the Indian Ocean and is thereby a part of the global heat conveyor belt. Heat and fresh water balance in the upper Pacific and Indian Oceans are only two aspects, where the ITF plays a major role [*Godfrey*, 1996]. Also, the ITF may be of importance for the heat supply feeding the deep water formation in the North Atlantic [*Godfrey*, 1996], which is considered to also have an influence on long-term climate variations.

Geological changes are expected to have modified the ITF in the last million years. Three to five million years ago, Earth was in the epoche of the Pliocene. The Pliocene is characterized by the linking of North and South America, a global temperature that was two to three degrees higher than today, a global sea level that was 25 meters higher and the formation of the Arctic ice cap. Most importantly, the Indonesian Passage in the Pliocene was wider and extended farther southwards, than it does today [*Jochum et al.,* 2009]. Research has found that the change of the Indonesian Passage to the present narrower channel was accompanied by a larger ITF transport [*Jochum et al.,* 2009]. In the Pliocene the ITF transport was smaller though the channel was wider. This stands in contrast to the fact that calculations, using Godfrey's Island Rule predict a larger pliocene ITF transport, due to stronger trade winds present at more southerly latitudes.

The tectonic changes of the Pliocene have also led to the large zonal sea surface temperature gradient in the equatorial Pacific, that we have today. It has been researched [*Jochum et al.,* 2009], that with the wide Indonesian Passage of the Pliocene, not only the total transport, but also the inflow of specifically north Pacific waters was smaller. To a higher degree, North Pacific water was caused to retroflect and stay in the Pacific in the Pliocene. This led to an increased eastward transport of warm North Pacific surface waters. Like this, the zonal sea surface temperature gradient in the equatorial Pacific used to be smaller. It is researched that as a result, the El Niño Southern Oscillation period was longer and of smaller amplitude in the Pliocene [*Jochum et al.,* 2009].

Consequently, the understanding of the ITF and the estimation of its total transport are of great interest, in order to for example understand and predict El Niño Southern Oscillation's behavior.

In this paper the ITF transport will be estimated using the Island Rule and the results will be compared with those of a reference numerical simulation, using the simulation program Veros. The validity of the Island Rule will be tested for different surface wind stress values in the Southern Ocean. The paper takes you through a theoretical section, explaining the Island Rule and the ITF behavior in general. The setup section deals with the way the ITF is simulated and examined. Also, it presents the code for the Island Rule computations. The test results will be presented and evaluated.

2 Theory

2.1 The Indonesian Through Flow (ITF)

Various estimates of the ITF total transport have been carried out over the years. Its estimation is difficult, due to the Indonesian channel system being complex. Furthermore, its allocation along both sides of the equator leads to possibly complex dynamics. This is due to varying Coriolis parameters. Also, currents of same magnitude close to the entrance of the passage make estimating the ITF transport difficult. These currents are the South Equatorial Current turning via the Halmahera Eddy to join the North Equatorial Counter Current and the Mindanao current turning via the Mindanao Eddy (see figure (1)). The Mindanao current is a current flowing along the eastern side of the southern Philippines. Seasonal and tidal variations hamper the estimation as well [*Godfrey*, 1996].

One of the main channels of the ITF is the Makassar Strait [*Godfrey*, 1996]. Further, the Lombok Strait and Timor Sea lead the way into the Indian Ocean (see figure (2)).

Modeling the ITF as a Sverdrup model with a wide Indonesian Passage results in a stagnant flow next to the eastern side of the passage [*Godfrey*, 1996]. In a Sverdrup model, horizontal pressure gradients are balanced by the horizontal Coriolis force and surface wind stresses. Further, it is assumed that the horizontal velocities and the horizontal pressure gradient vanish at a moderate depth of the ocean and that the system is baroclinic [*Sverdrup*, 1947].

A way of estimating the ITF transport is by using the Island Rule.

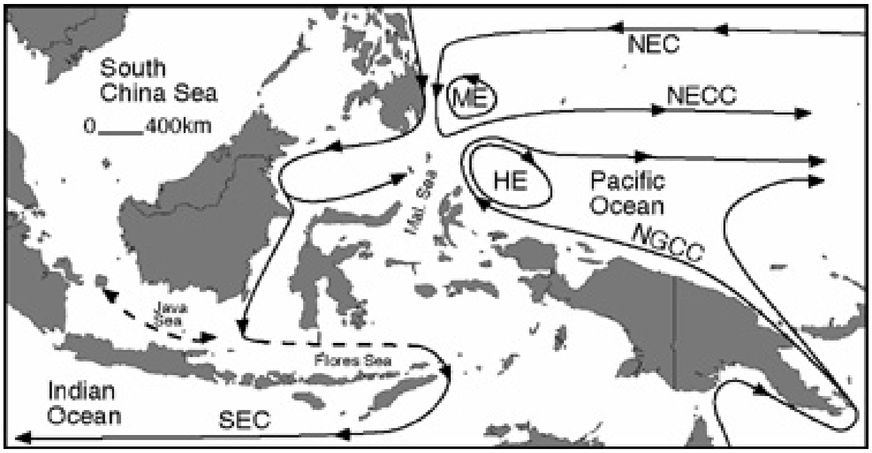

Figure 1: **Currents in the equatorial east Pacific:** The northern equatorial current (NEC) and the New Guinea Costal Current (NGCC) or South Equatorial Current (SEC) turn eastward via the Mindanao Eddy (ME) and Halmahera Eddy (HE) respectively to join the North Equatorial Counter Current (NECC). The ITF is advected in the Indian Ocean with in the SEC.

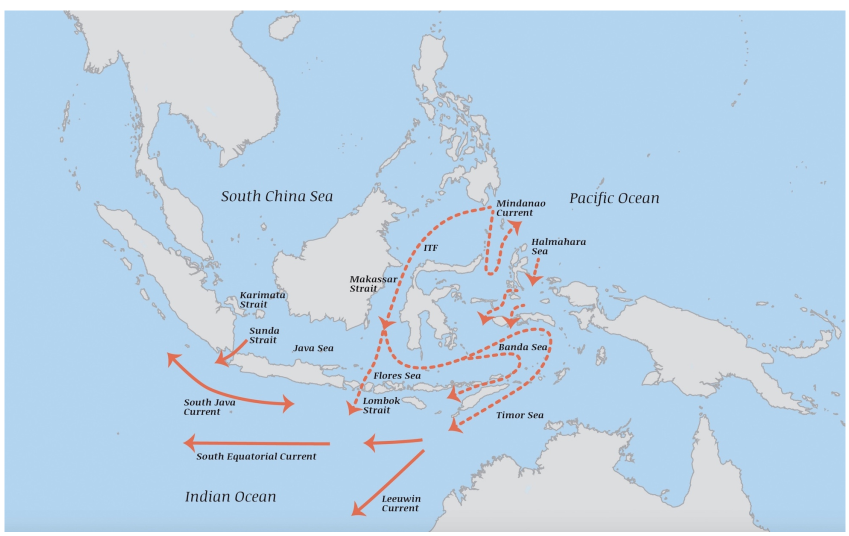

Figure 2: **Water flows in the Indonesian Passage:** Main pathways of the ITF are the Makassar Strait, Lombok Strait and Timor Sea.

2.2 The Island Rule

The Island Rule, proposed by John Stuart Godfrey in 1989, is used to examine the water transport between an island and the external boundaries of the ocean basin in which it is placed. The transport around the island is the same at any point when the flow has reached a steady state circulation. The circulation is assumed to be two-dimensional and the system is assumed to be barotropic. A streamfunction, ψ, is introduced. The streamfunction values must be constants along the boundaries of the ocean basin and the island, due to the impossibility of flow across these boundaries. One of these constants can be set individually. If the total transport around the island is wanted to be found directly, it is necessary to set $\psi = 0$ at the basin's outer boundary. This way, the total transport around the island will be given by ψ at the island's boundaries [*Pedlosky,* 1997].

In order to find ψ, the circulation along the island boundaries has to be considered. Dealing with the circulation at the eastern boundary of the island can become difficult though, due to a large dissipation on the eastern side. This large dissipation is caused by westward intensification. Westward intensification is the process by which in a barotropic homogeneous rectangular ocean, flow tends to be concentrated at the western boundary of the ocean [*Stommel,* 1948]. In the research, the flow is assumed to be geostrophic and the basin is situated along the equator. A linear drag and wind stress in the horizontal Navier-Stokes equation are considered. When introducing the beta plane, which is a linearly varying Coriolis parameter, a westward intensification of the flow is observed [*Stommel,* 1948]. This leads to large dissipation at the eastern boundary of the island.

Because of the difficulties related to the westward intensification, an altered contour around the island was suggested (see figure(3)). This contour avoids the eastern boundaries of the island. Instead the contour runs from the upper and lower extremities of the island along the corresponding northern and southern latitudes eastwards, where it closes at the eastern boundary of the ocean basin (see figure(3)) [*Pedlosky,* 1997]. The dynamical equivalence of these two contours was shown by Godfrey [*Pedlosky,* 1997].

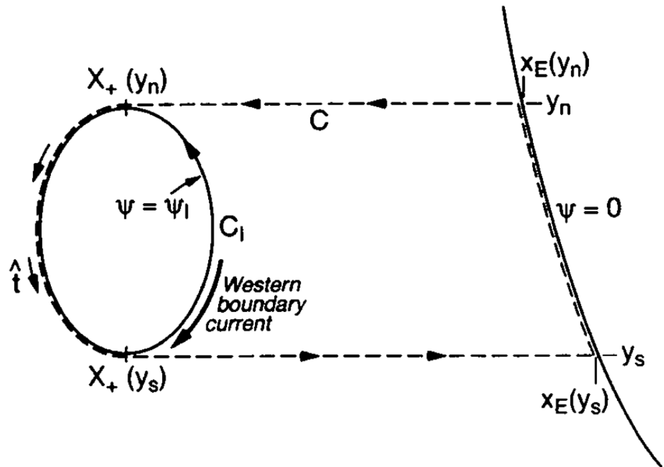

Figure 3: **Island Rule contour:** Contour suggested by J.S. Godfrey for the Island Rule. The contour runs from the western boundary of the island over its extremities ($X_+(y_n)$, $X_+(y_s)$) zonally across the basin to the eastern boundary of the basin ($x_E(y_n)$, $x_E(y_s)$), where it closes. Note the avoidance of western boundary currents.

The Island Rule is presented as follows:

$$\psi = -\oint \vec{\tau} \cdot \vec{dl} / [\rho_0 (f(y_n) - f(y_s))], \tag{1}$$

where τ is the surface wind stress. The long-path component of the surface wind stress is integrated along the contour (see figure (3)). The integral is divided by the term ($[\rho_0(f(y_n) - f(y_s))]$), where ρ_0 is the density of the seawater and $f(y_n)$ and $f(y_s)$ are the Coriolis parameters at the northern and at the southern latitude, respectively.

The total transport, ψ, yielded by the Island Rule will be found around Australia, which in turn equals the ITF transport, due to the geographical vis-à-vis allocation of Australia and the Indonesian islands.

3 Setup

In order to validate the ITF transport as yielded by the Island Rule, ψ_{IR}, the Island Rule results willbe compared with those of an acknowledged numerical simulation model ($\psi_{Simulation}$).

3.1 Simulation

An ocean simulation program called Veros is used. In this program, it is possible to set-up wind stresses among many other properties and compute the ITF transport accordingly. Example model setups are provided. The program is written in pure python and it is an open source program.

The model setup used here is the `global_four_degree_ITF` setup provided by the Niels Bohr Institute. In the model, precomputed settings are read in from a NetCDF data file (`Open_ITF_4deg.nc`). By the data file, initial values for properties such as salinity, temperature, bathymetry and surface wind stresses are specified for the simulation (see Appendix).

The numerical results of the simulations are provided in the time domain and in the model domain of an Arakawa C-grid. This grid is staggered in all three spacial dimensions (see figure (4)).

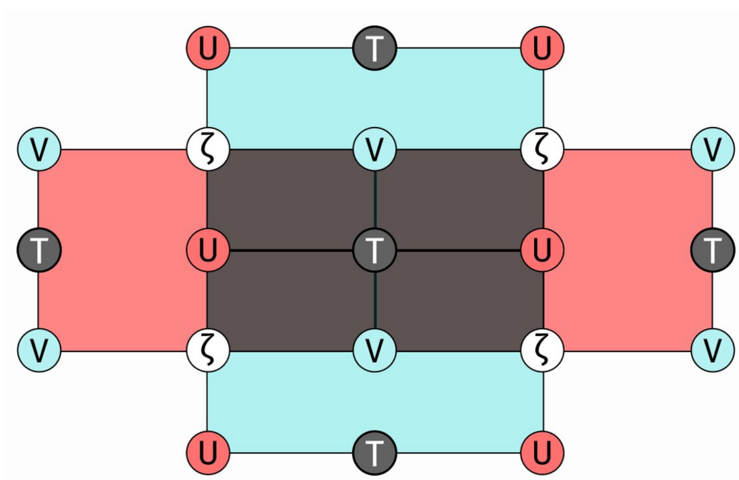

Figure 4: **Structure of Arakawa C-grid:** Grid often used in oceanography or meteorology. Zonal velocities u are computed at the centers of the left and right grid faces. Computation of meridional velocity v takes place at the centers of the upper and lower grid faces. Calculation of tracers (T) take place at the center of the grid.

Salinity and temperature are two examples for tracers. These are computed in the center of the grid cell. Zonal (u), meridional (v) and vertical velocities are calculated at points 'U', 'V' and 'W' (see figure (4) ('W' is not depicted)).

The `global_four_degree_ITF` model considers the globe in total and an open Indonesian passage is implemented. The grid is arranged with a four degree spacing (see figure (5)). Because of the Arakawa C-grid, two sets of longitudes and latitudes are given. There are 90 longitude and 40 latitude divisions, together reaching from 0°to 360°in longitude and from -80°to +80°in latitude. This is a result of the 4 °spacing. There are 15 divisions in the vertical dimension.

Due to this resolution, the islands of Australia and Papua New Guinea are joint together to Australasia. The rest of the Indonesian islands, the Philippines, Borneo, and further smaller islands located in the Indonesian seas form one unit together, as well (see figure (5)). The channel situated between this unit and Australasia represents the Indonesian Passage.

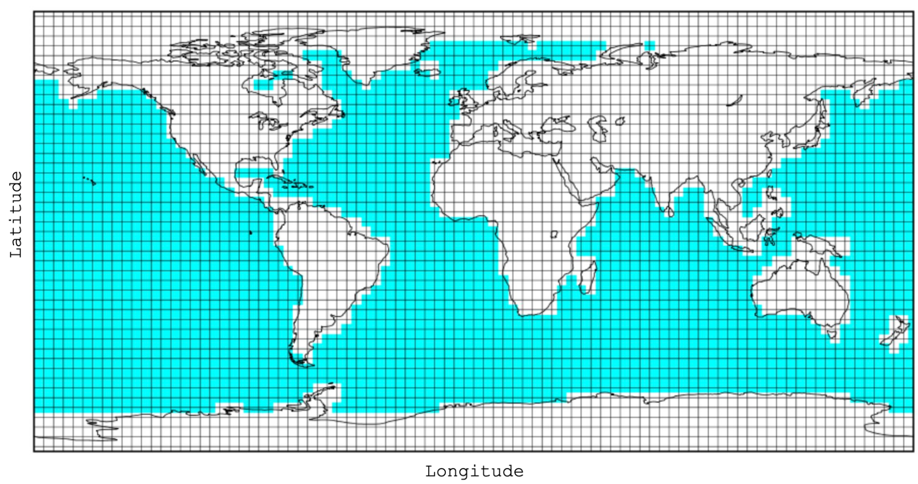

Figure 5: **global_four_degree_ITF grid:** 90 division for the longitudes are given and 40 for the latitudes. This resolution results in a ragged contour between land (white) and sea (blue). Actual topography is contoured in black.

For each simulation a time-length of the run must be set, as well as the tracer sample period. Computed variable values, such as tracers, velocities, meridional and total transports are saved into a `4deg.snapshot.nc` file for each one-year time period.

The surface wind stresses remain constant throughout each simulation. A default surface wind stress is given by the `Open_ITF_4deg.nc` file (see figure (8)). The surface wind stresses are changed manually for each run. One simulation with globally zero wind stress is performed. With this, a basic transport with absent wind can be detected. For all other runs, all zonal surface wind stresses are varied in the Southern Ocean below a -32°S latitude. This specific location was chosen, in order to satisfy the wind stress' continuity. At -32°S the zonal surface wind stress equals approximately zero along the whole latitude (see figure (6)). Therefore varying the wind stress below this latitude retains the continuity. The wind stresses are varied by factors between 0 and 2, including the default wind stress ($factor = 1$). The change of wind stress is achieved in the `global_four_degree_ITF` setup by subclassing the forcing setting functions. In total, 13 different surface wind stresses have been simulated.

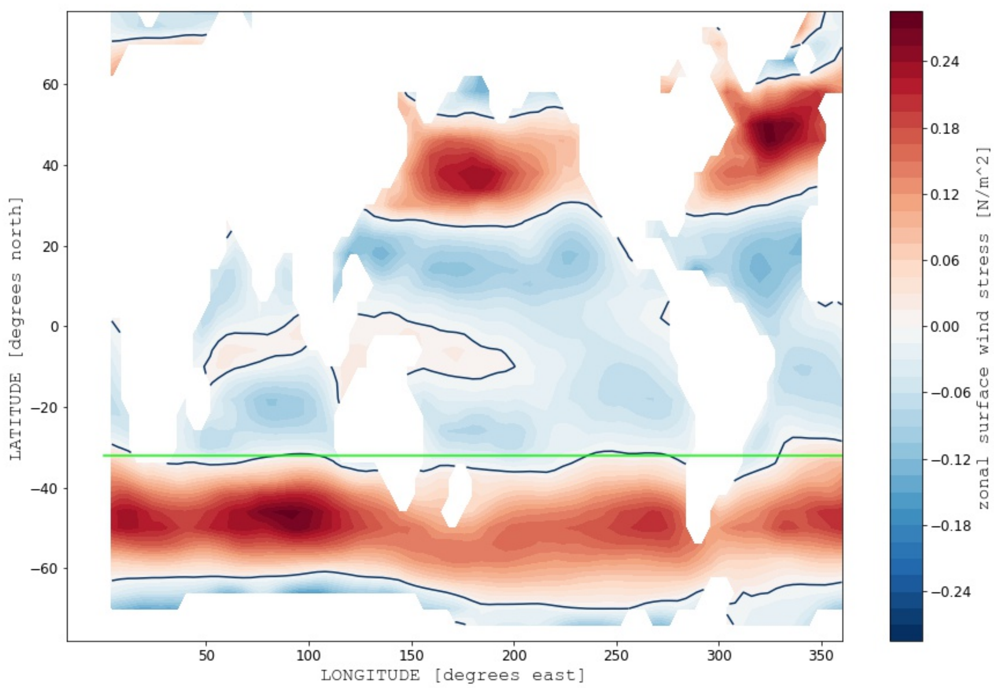

Figure 6: **Zonal wind stresses:** Zonal surface wind stresses are shown (see contour). The zero zonal surface wind stress line is contoured extra (dark blue line contour). The -32°S latitude is plotted (light green line).

The streamfunction, $\psi_{Simulation}$ is computed throughout the run time and saved into the `4deg.snapshot.nc` file. Positive streamfunction values indicate a total clockwise motion of the water, whereas negative values indicate a counterclockwise movement.

In order to ensure that a steady state circulation is reached, the ITF transport is plotted as a time series. In this plot, it will be validated that the ITF approaches an asymptote. A minimum run-length for the ITF to reach a steady state is determined. The results for $\psi_{Simulation}$ will be compared to the results yielded by the Island Rule (ψ_{IR}) for different wind stresses.

3.2 Island Rule Estimation

In order to compute the ITF transport, ψ_{IR}, using the Island Rule, a contour has to be established. This contour has to avoid western boundary currents (see figure (3)). In order for this to be satisfied in our setup, a contour is chosen that runs along the eastern coasts of Australasia and South America (see figure (7)) and runs along the two latitudes of -2 °S and -46°S.

The contour is divided into four parts in order to enable the analysis of each part's contribution to the total integral. The parts are shown in figure (7) and will in the following be called: equatorial ocean, Southern Ocean, western Australia and South American part, each.

The horizontal and vertical distances between each point are all set equally to 444710 meters, taken from the simulation grid. The wind stresses computed by the simulation and saved in the `4deg.snapshot.nc` files are extracted and used for the integral. Default wind stresses are shown in figure (8). Wind stresses along the Southern Ocean part of the path are the strongest. This is due to the particularly strong prevailing westerly winds circulating Antarctica. The strength is due to little topography in the southern hemisphere. At the equator the weak easterly trade winds are prevailing. The region is also referred to as the doldrums, due to its weak wind characteristics. Also, figure (8) illustrates the westward intensification of the ocean flow (see streamfunction contour).

The sea water density and the coriolis parameters ρ_0, f_N and f_S (see equation (1)) are specified as: $\rho_0 = 1020\frac{kg}{m^3}$, $f_N = 2 * \frac{2*\pi}{24*60^2} * sin(-2°) = -5.08e^{-6}$ and $f_S = 2 * \frac{2*\pi}{24*60^2} * sin(-46°) = 1.05e^{-4}$.

The complete implementation can be found under:
`https://github.com/cecilieboy/Godfrey-Island-Rule/blob/master/Island_Rule.py` or in the Appendix.

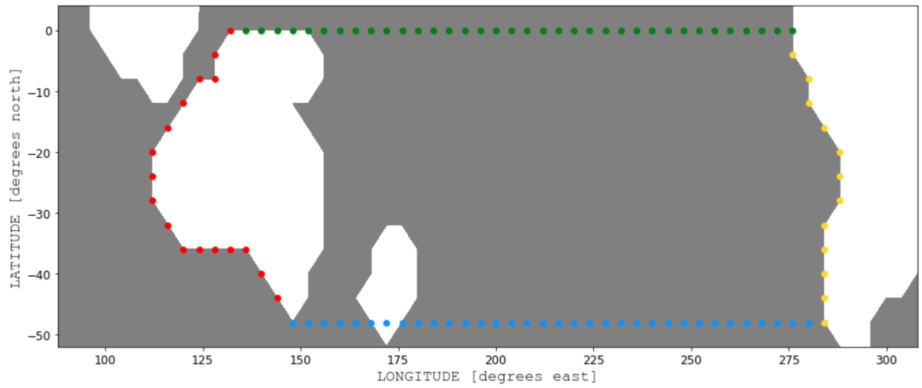

Figure 7: **Contour utilized for ITF transport computation by the Island Rule:** The contour consists of four parts: equatorial ocean part, located on -2 °S latitude (green), Southern Ocean part at -46°S (medium blue), west Australian part running along the west coast of Australia (red) and South American part running along the western coast of South America (yellow).

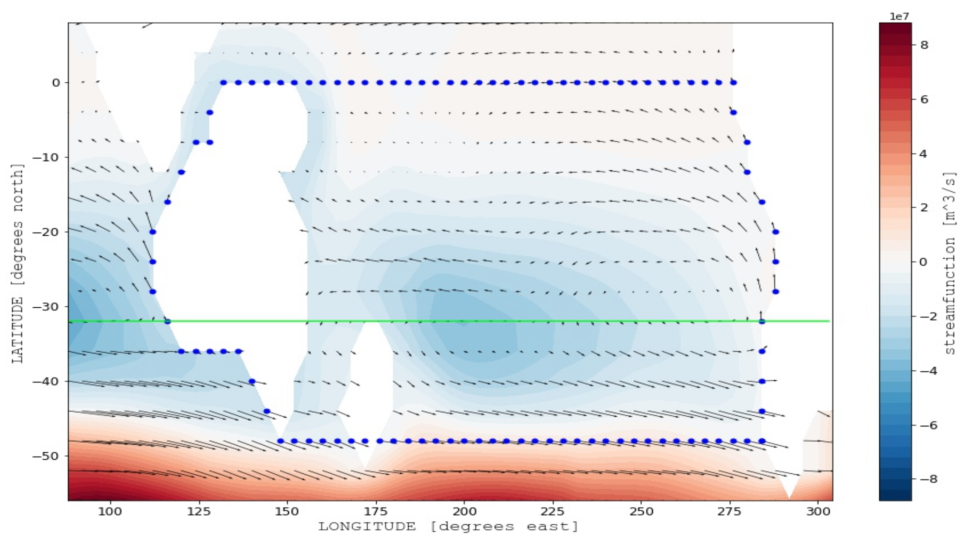

Figure 8: **Default surface wind stresses:** Wind stresses for $factor = 1$ are shown as black vectors indicating the direction and relative strength. The corresponding streamfunction is also shown (see contour). Note the westward intensification shown by the streamfunction. The contour used for the Island Rule computation is indicated (blue points), as well as the -32°S latitude (light green line).

4 Results and Discussion

4.1 Initial Examination of the Simulated ITF

In order to get a feeling for the flow through the Indonesian Passage as generated by the simulation, an initial run for the default wind stress is performed and the results are plotted after a run length of 200 years. All velocity vectors present a south westward directed flow (see figure (9)). The velocity vector at the eastern bay is directed normal to the coast (see figure (9)). This indicates a stagnant flow along the eastern boundary of the channel, as predicted by the Sverdrup model.

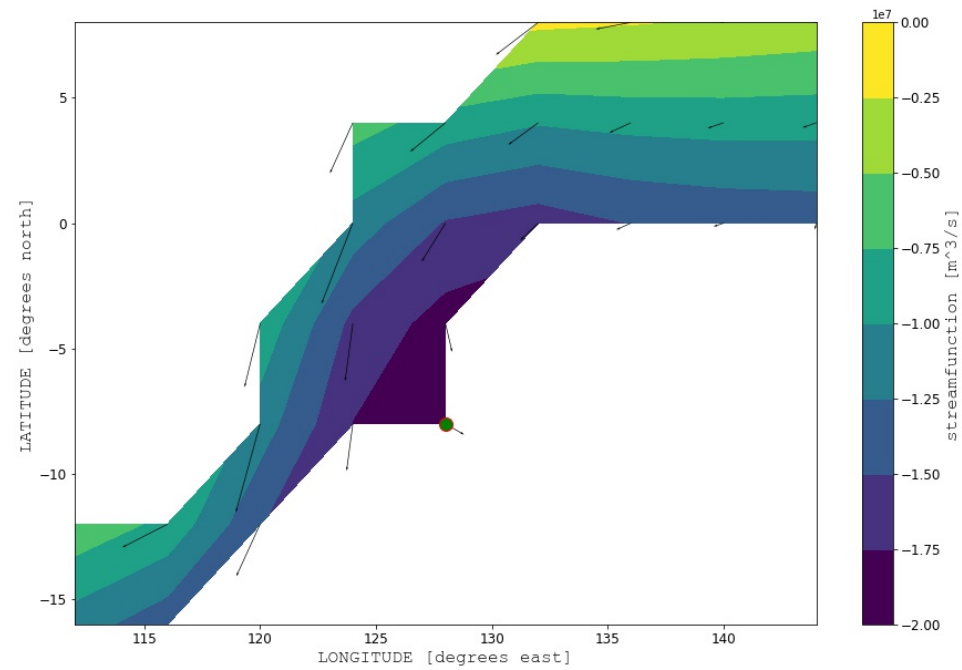

Figure 9: **Streamfunction in the Indonesian Passage at default wind stress:** It is zoomed onto the Indonesian Passage. Velocity vectors are shown (black arrows) along with the streamfunction (see contour). The ITF transport, $\psi_{Simulation}$, is given by the streamfunction value at 128°E, -8°S (green point with red edge).

All flows indicate a counterclockwise movement (see figure (9)). The total transport is directed from the Pacific into the Indian Ocean, as expected. Generally, flows at the eastern boundary are stronger than at the western boundary.

For simplicity reasons, the cell situated in the Indonesian Passage with the highest streamfunction value is selected to represent the ITF transport. Because of this, the ITF transport is found to be represented by the streamfunction value at 128 °E, -8 °S.

4.2 Steady State Condition

When the wind stress is set to zero, globally, a minimum run time of 150 years is needed for the transport to reach a steady state (see figure (10)). The ITF transport approaches an asymptote of approximately $-8.5Sv$.

With the default wind stress ($factor = 1$), a minimum run time of 300 years is needed (see figure (11)). For reduced wind stresses ($factor < 1$), run times of minimum 150 years are sufficient. For increased wind stress factors ($factor > 1$) run times of 350-400 years are necessary for a steady state to be reached. Thus, the larger the wind stress factor, the larger is also the relaxation time (see figure (11)). Generally, smaller wind stress factors approach an asymptote to a higher degree than larger wind stress factors.

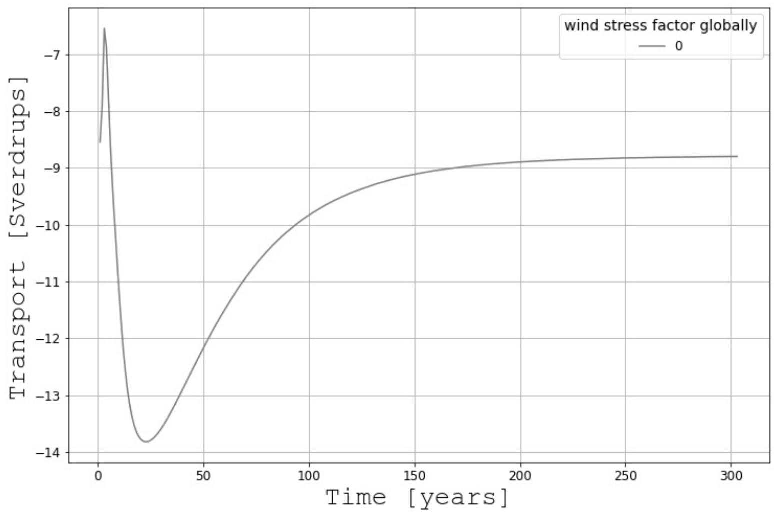

Figure 10: **Time series of the ITF transport for no wind stress, globally.**

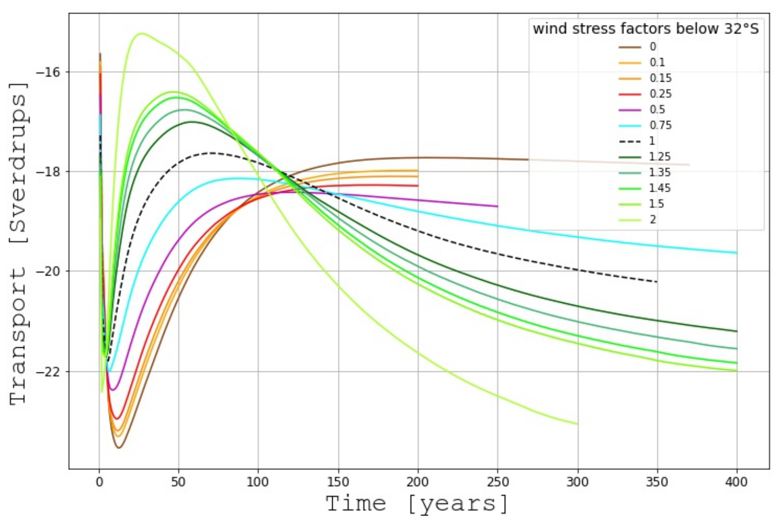

Figure 11: **Time series of the ITF transport for different wind stress factors.**

4.3 Test Results

Not surprisingly, the ITF transport, $\psi_{Simulation}$ increases with an increasing wind stress factor. This also applies to ψ_{IR} (see table (1)).

Wind stress (factor)	**Time** *[years]*	ψ_{IR}[Sv]	$\psi_{Simulation}$[Sv]
globally 0	303	0	-8.8
(below -32 °S):			
0.0	370	-3.5	-17.8
0.10	200	-5.6	-17.9
0.15	200	-6.6	-18.1
0.25	200	-8.7	-18.3
0.50	250	-13.8	-18.7
0.75	400	-18.9	-19.6
1.0 (default)	350	-23.9	-20.2
1.25	400	-29.1	-21.2
1.35	400	-31.1	-21.5
1.45	400	-33.1	-21.8
1.5	400	-34.2	-21.9
2.0	350	-44.4	-23.07

Table 1: **Transports yielded by the Island Rule and by simulations** : Transports are shown along with their corresponding wind stress factors and the run time of the simulation.

When the wind stress is set to be gloablly zero, a non-zero ITF transport is observed in the simulation (see table (1)). According to the Island Rule no ITF transport is expected. The resulting relative error is 100% (see figure (14)). The fact that a globally zero surface wind stress still leads to an ITF indicates further driving factors of the Indonesian through flow other than the wind stress.

For the default wind-stress factor, the Island Rule overestimates the ITF transport, compared to the simulated transport. Simulations yield a value $\psi_{Simulation} = -20.2\ Sv$, whereas according to the Island Rule, a transport of $-23.9\ Sv$ would be expected. This leads to a relative error of 18.6%.

For wind stress factors minor than one, the Island Rule underestimates the simulated ITF transport (see figure (12)). The amount by which the simulated transport is underestimated increases, the smaller the wind stress factor. Due to this, the relative error increases the smaller the wind stress factor is (see figure (14)).

For wind stress factors larger than one, the ITF transport is overestimated. The larger the wind stress factor the bigger is the deviation from the simulated value (see figure (12)) and the larger is also the relative error (see figure (14)).

Around wind stress factors of 0.75 and the default wind stress factor 1 the smallest relative errors are found. The Island Rule transports ψ_{IR} are more sensible towards wind changes than the simulated ITF transports $\psi_{Simulation}$.

The four parts of the Island Rule contour contribute to the total estimation of the ITF to different extends (see figure (13)). As expected, the Southern Ocean winds have the greatest impact on the total integral. This is because the wind has been changed here and because the wind stress component along the contour is generally the largest here. Further, the western Australia integral has a little impact (see figure (13)), stemming from the fact that this contour dips into the -32°S zone and contributes with a certain long-path component, as well. The equatorial ocean integral remains constant, due to no wind changes and has a little value, due to weak trade winds at the equator, as expected. The South American integral part varies very little, as the vectors have small long-path components below -32 °S. Thus, the main contributions to the change of ITF transport estimations by the Island Rule result from the Southern Ocean wind stresses, as expected, and from wind stresses at the west Australian coast.

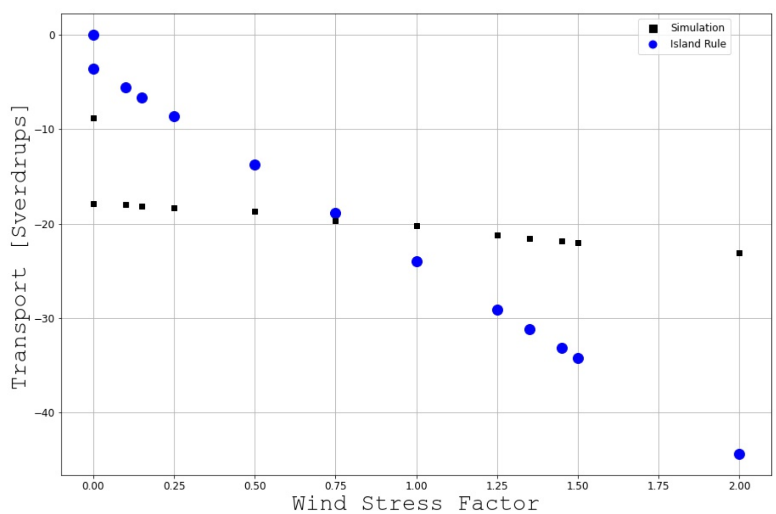

Figure 12: **ITF transports yielded by the Island Rule and by simulations for different wind stress factors.**

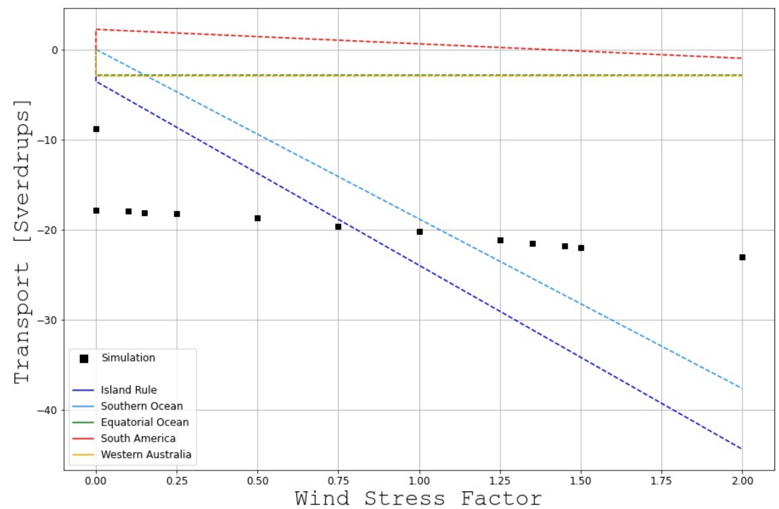

Figure 13: **ITF transports and the contributions of the integration parts.** The total integration is plotted as a dark blue line. For the contribution description, see legend.

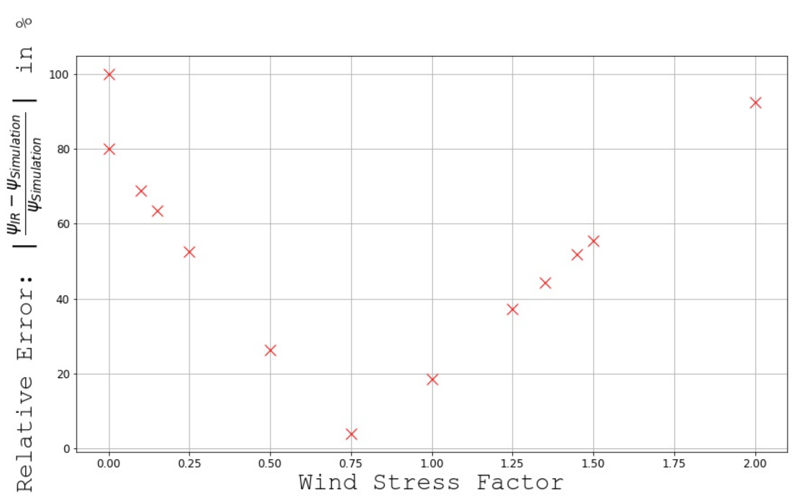

Figure 14: **Relative error in the estimation of the ITF transport by the Island Rule.**

As a result, the Island Rule estimations of the ITF trasport reveal a dependence on the surface wind stress which is too large compared to the results obtained from Veros simulations. Other factors driving the ITF must be considered and are suggested in the following.

4.4 Further ITF Driving Forces

An expansion of the Island Rule was later presented by Godfrey [Godfrey, 1996]:

$$
\begin{aligned}
T = &- \oint \vec{\tau} \cdot \vec{dl} / [\rho_0(f(y_n) - f(y_s))] \\
&- Z[p_{\text{West Tasmania at -44 °S}}(-Z) - p_{\text{West South America at -44 °S}}(-Z)] / [\rho_0(f(y_n) - f(y_s))] \\
&- \int_{-Z}^{0} \Delta p_{NZ}(z) dz \} / [\rho_0(f(y_n) - f(y_s))] \\
&- \int_{\text{Indonesian Passage}} F(l) dl / [\rho_0(f(y_n) - f(y_s))] \\
&+ \int_{\text{Southern Ocean along -44 °S}} f V_p dl / [\rho_0(f(y_n) - f(y_s))] \\
&- \int_{\text{Whole contour except Southern Ocean}} NL dl / [\rho_0(f(y_n) - f(y_s))] \\
&+ W,
\end{aligned} \tag{2}
$$

where $p_{\text{West Tasmania at -44 °S}}(-Z)$ and $p_{\text{West South America at -44 °S}}(-Z)$ are the pressures at the Indonesian sill depth Z, where $Z = 1500m$. Δp_{NZ} is the pressure difference from east to west across New Zealand. F is friction and V_p is the depth-integrated flow component perpendicular to the contour. Nonlinear terms are abbreviated as NL and W is the upward vertical transport through the sill depth in the Pacific Ocean.

In this paper, the first term of equation (2) was investigated (see equation (1)). However, nonlinearities are found to have an important impact on the ITF transport, as well [Jochum et al., 2009]. Increased nonlinearities lead to reduced ITF transports. Thus, the possibility of increased nonlinearities has to be investigated in further research.

The Veros simulation model calculates the meridional overturning and returns it in the NetCDF formatted file `4deg_overturning.nc`. Plotting the `vsf_depth` data from the overturning file shows the meridional transport in a meridional section, representing a zonally averaged picture. By plotting the transport in a certain depth for varying Southern Ocean wind stresses, a feeling for the role of the wind for the surface and deep-water flow is obtained. Figure (15) shows that meridional transport is present, also when there is no surface wind stress present. This may serve to explain the non-zero ITF transport, obtained by the simulation for globally zero wind stress. The meridional transports are of values similar to the simulated ITF transport ($-8.8SV$). It is to be remarked though, that the meridional transport is not independent of the wind stress, as shown by figure (15).

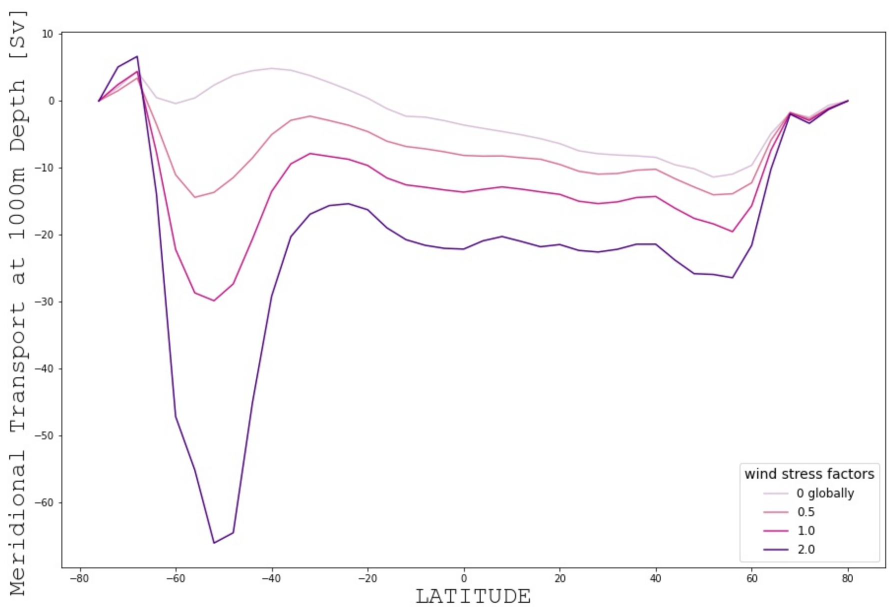

Figure 15: **Meridional transport at 1000 meter depth for different wind stress factors**.

Finally, the approximation of the ITF transport being described by a two dimensional streamfunction is to be regarded critically. By introducing stratification, baroclinicity can be taken into account. Baroclinical flows may possibly play an important role for the ITF transport.

5 Conclusion

Generally, the ITF transport increases, when Southern Ocean wind stresses are increased in their zonally eastwards direction. However, the ITF transport yielded by the Island Rule shows more sensible behavior towards wind variation than the ITF transports yielded by the simulations. Moreover, with no wind stress present at all, a non-zero ITF transport is observed by the simulation. Other factors must be considered as driving forces for the Indonesian through flow, if the simulatinos are accurate.

Possible factors were identified to be nonlinearities and meridional transport. A present meridional transport is found to explain the non-zero transport through the Indonesian Passage for no surface wind stress present at all. In further research, the interconnection of the surface wind stress and the meridional transport should be analyzed more precisely. Finally, the difficulties in approximating the ITF transport by a two dimensional, barotropic streamfunction are to be considered. Baroclinical currents are suggested to be of importance.

References

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A Appendix

Link for implementation:
https://github.com/cecilieboy/Godfrey-Island-Rule/blob/master/Island_Rule.py.

Code for the the estimation of the ITF transport by the Island Rule:

```
#%% #import packages

import xarray as xr
import numpy as np
import pandas as pd
from matplotlib import pyplot as plt

#%%
def get_indices():
    '''
    Funciton defines longitudes and latitudes for each part of the path.
    There are four parts: Southern Ocean, Equatorial Ocean, South America
    and Western Australia (so, eo, sa, wa). A random snapshot file is opened.
    Indices are extracted in a double loop from the ds.xu/ds.xt and
    ds.yu/ds.yt files. Function returns these indices.
    '''

    ds = xr.open_dataset('4deg_000.snapshot.nc')

    so_y = np.full(35,-46)
    so_x = np.array([148, 152, 156, 160, 164, 168, 172, 176, 180, 184, 188,
                     192, 196, 200, 204, 208, 212, 216, 220, 224, 228,
                     232, 236, 240, 244, 248, 252, 256, 260, 264, 268,
                     272, 276, 280, 284])
    i_so_y = []
    i_so_x = []
    for i in range (len(so_y)):
        for j in range (len(ds.yt)):
            if so_y[i] == ds.yt[j]:
                i_so_y.append(j)
    for i in range (len(so_y)):
        for j in range (len(ds.xu)):
            if so_x[i] == ds.xu[j]:
                i_so_x.append(j)
```

```
eo_y = np.full(36,-2)
eo_x = np.array([276, 272, 268, 264, 260,
                    256, 252, 248, 244, 240, 236, 232, 228, 224, 220, 216,
                     212, 208, 204, 200, 196, 192, 188, 184, 180, 176, 172,
                     168, 164, 160, 156, 152, 148, 144, 140, 136])
i_eo_y = []
i_eo_x = []
for i in range (len(eo_y)):
    for j in range (len(ds.yt)):
        if eo_y[i] == ds.yt[j]:
            i_eo_y.append(j)
for i in range (len(eo_y)):
    for j in range (len(ds.xu)):
        if eo_x[i] == ds.xu[j]:
            i_eo_x.append(j)



sa_x = np.array([284, 284, 284, 284, 288, 288, 288, 284, 280, 280, 276])
sa_y = np.array([-44, -40, -36, -32, -28, -24, -20, -16, -12, -8, -4])
i_sa_y = []
i_sa_x = []
for i in range (len(sa_y)):
    for j in range (len(ds.yu)):
        if sa_y[i] == ds.yu[j]:
            i_sa_y.append(j)
for i in range (len(sa_y)):
    for j in range (len(ds.xu)):
        if sa_x[i] == ds.xu[j]:
            i_sa_x.append(j)



wa_x = np.array([132, 128, 128, 124, 120, 116, 112, 112, 112, 116, 120,
                     124, 128, 132, 136, 140])
wa_y = np.array([0, -4, -8, -8, -12, -16, -20, -24, -28, -32, -36, -36,
                     -36, -36, -36, -40])
i_wa_y = []
i_wa_x = []
for i in range (len(wa_y)):
    for j in range (len(ds.yu)):
        if wa_y[i] == ds.yu[j]:
            i_wa_y.append(j)
for i in range (len(wa_y)):
    for j in range (len(ds.xu)):
        if wa_x[i] == ds.xu[j]:
```

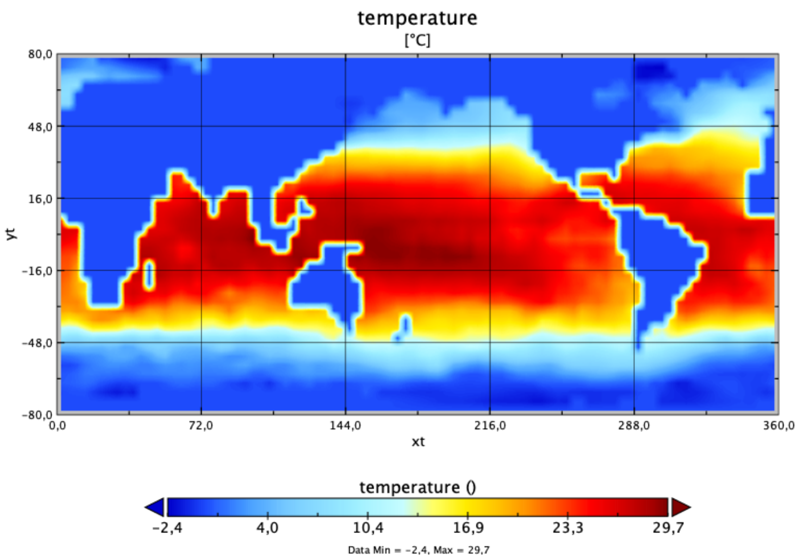
```python
                i_wa_x.append(j)



    return i_so_x, i_so_y, i_eo_x, i_eo_y, i_sa_x, i_sa_y, i_wa_x, i_wa_y


#%%
def Island_Rule(wind, i_so_x, i_so_y, i_eo_x, i_eo_y, i_sa_x, i_sa_y, i_wa_x, i_wa_y):
    ''' Function opens a nc snapshot file given by the wind input. NC snapshot
    files should be named as 4deg_{'inser wind factor'}.snapshot.nc. The value of
    the throughflow from the simulation is extracted in the point 128e, -8s.
    Integration for each part is done in an integration loop, integrating the
    surface wind stress in the correct direction. Where the points are located
    on land a windstress of zero is added. All ds.dxu/ds.dxt values are the same,
    this also applies to all ds.dyu/ds.dyt values, so the first is just taken
    (ds.dxu[0], ds.dyu[0]). The sum of the part integrations is taken and divided
    by the constants as defined in the Island Rule. Function returns the Island Rule
    integral given in Sverdrups, the simulated throughflow in Sverdrups, the
    contribution to the integral of each part of the path in Sverdrups and the
    relative Error of the Island Rule compared to the simulated throughflow.
    '''

    #defining constants:
    f_2s = 2 * (2 * np.pi / (24 * 60 * 60)) * np.sin(-(1/90) * np.pi)
    f_44s = 2 * (2 * np.pi / (24 * 60 * 60)) * np.sin(-(23/90) * np.pi)
    rho = 1020

    #opens snapshot file for specific windstress
    ds = xr.open_dataset('4deg_{}.snapshot.nc'.format(wind))

    #Throughflow result of simulation is found (128e, 8s):
    psi = ds.psi[-1, 17, 31]




    #Integral along Southern Ocean:
    SO = 0
    for i in range(len(i_so_y)):
        if i == 4 or i == 5 or i ==6:
            SO += 0
        else:
            SO += ds.surface_taux[-1, i_so_y[i], i_so_x[i]] * ds.dxu[0]
```

```python
#Integral along Equatorial Ocean:
EO = 0
for i in range(len(i_eo_y)):
    EO += - ds.surface_taux[-1, i_eo_y[i], i_eo_x[i]] * ds.dxu[0]


#Integral along South America:
SA = 0
for i in range(len(i_sa_y)):
    if i < 3:
        SA += ds.surface_tauy[-1, i_sa_y[i], i_sa_x[i]] * ds.dyu[0]
    if i == 3:
        SA += (ds.surface_tauy[-1, i_sa_y[i], i_sa_x[i]] * ds.dyu[0]
               + ds.surface_taux[-1, i_sa_y[i], i_sa_x[i]] * ds.dxu[0])
    if i == 4 or i == 5:
        SA += ds.surface_tauy[-1, i_sa_y[i], i_sa_x[i]] * ds.dyu[0]
    if i == 6 or i == 7:
        SA += (ds.surface_tauy[-1, i_sa_y[i], i_sa_x[i]] * ds.dyu[0]
               - ds.surface_taux[-1, i_sa_y[i], i_sa_x[i]] * ds.dxu[0])
    if i == 8:
        SA += ds.surface_tauy[-1, i_sa_y[i], i_sa_x[i]] * ds.dyu[0]
    if i == 9:
        SA += (ds.surface_tauy[-1, i_sa_y[i], i_sa_x[i]] * ds.dyu[0]
               - ds.surface_taux[-1, i_sa_y[i], i_sa_x[i]] * ds.dxu[0])
    if i == 10:
        SA += ds.surface_tauy[-1, i_sa_y[i], i_sa_x[i]] * ds.dyu[0]


#Integral along western Australia:
WA = 0
for i in range (len(i_wa_y)):
    if i == 0:
        WA += (- ds.surface_tauy[-1, i_wa_y[i], i_wa_x[i]] * ds.dyu[0]
               - ds.surface_taux[-1, i_wa_y[i], i_wa_x[i]] * ds.dxu[0])
    if i == 1:
        WA += - ds.surface_tauy[-1, i_wa_y[i], i_wa_x[i]] * ds.dyu[0]
    if i == 2:
        WA += - ds.surface_taux[-1, i_wa_y[i], i_wa_x[i]] * ds.dxu[0]
    if i == 3 or i == 4 or i == 5:
        WA += (- ds.surface_tauy[-1, i_wa_y[i], i_wa_x[i]] * ds.dyu[0]
               - ds.surface_taux[-1, i_sa_y[i], i_sa_x[i]] * ds.dxu[0])
    if i == 6 or i == 7:
        WA += - ds.surface_tauy[-1, i_wa_y[i], i_wa_x[i]] * ds.dyu[0]
    if i == 8 or i == 9:
        WA += (- ds.surface_tauy[-1, i_wa_y[i], i_wa_x[i]] * ds.dyu[0]
               + ds.surface_taux[-1, i_wa_y[i], i_wa_x[i]] * ds.dxu[0])
```

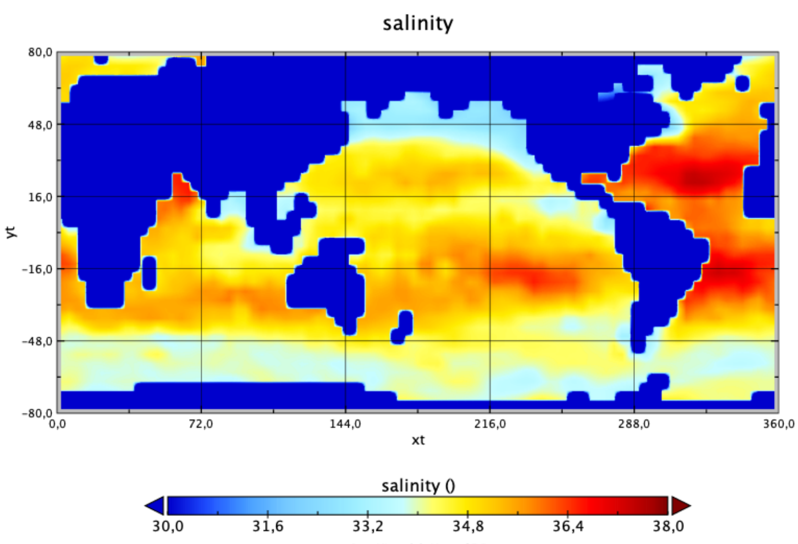
```python
        if i == 10 or i == 11 or i == 12 or i == 13:
            WA += ds.surface_taux[-1, i_wa_y[i], i_wa_x[i]] * ds.dxu[0]
        if i == 14 or i == 15:
            WA += (- ds.surface_tauy[-1, i_wa_y[i], i_wa_x[i]] * ds.dyu[0]
                    + ds.surface_taux[-1, i_wa_y[i], i_wa_x[i]] * ds.dxu[0])

    tau = SO + EO + SA + WA
    IR = - tau / (rho * (f_2s - f_44s))


    SO_c = (- SO / (rho * (f_2s - f_44s)))
    EO_c = (- EO / (rho * (f_2s - f_44s)))
    SA_c = (- SA / (rho * (f_2s - f_44s)))
    WA_c = (- WA / (rho * (f_2s - f_44s)))

    RELERR = np.abs((1 - (IR / psi)) * 100)
    return IR / 10**6, psi / 10**6, SO_c / 10**6, EO_c / 10**6, SA_c / 10**6, \
            WA_c / 10**6, RELERR
#%%
def call_Island_Rule(winds):
    '''
    Function calls up the Island_Rule function for all winds in order to
    assemble arrays of Island Rules, simulated throughflow values,
    contributions of paths and relative errors. Results are saved into
    a csv file.
    '''

    i_so_x, i_so_y, i_eo_x, i_eo_y, i_sa_x, i_sa_y, i_wa_x, i_wa_y = get_indices()

    Island_rules = np.zeros(len(winds))
    psis = np.zeros(len(winds))
    SO_cs = np.zeros(len(winds))
    EO_cs = np.zeros(len(winds))
    SA_cs = np.zeros(len(winds))
    WA_cs = np.zeros(len(winds))
    RELERRS = np.zeros(len(winds))
    for i in range(len(winds)):
        Island_rules[i], psis[i], SO_cs[i], EO_cs[i], SA_cs[i], WA_cs[i], RELERRS[i] = \
            Island_Rule(winds[i], i_so_x, i_so_y, i_eo_x, i_eo_y, i_sa_x, i_sa_y,
                        i_wa_x, i_wa_y)


    Data = {'winds': winds, 'islandrules': Island_rules, 'psis': psis,
            'southernOcean': SO_cs, 'equatorialOcean': EO_cs,
           'SA': SA_cs, 'WA': WA_cs, 'RELERR': RELERRS}
    df = pd.DataFrame(Data)
```

```python
    df.to_csv('solutions.csv')

    #return Island_rules, psis, SO_cs, EO_cs, SA_cs, WA_cs, RELERRS

#%%
if __name__ == '__main__':
    call_Island_Rule(['000', '0', '01', '015', '025', '05', '075',
                      '1', '125', '135', '145' ,'15', '2'])
```

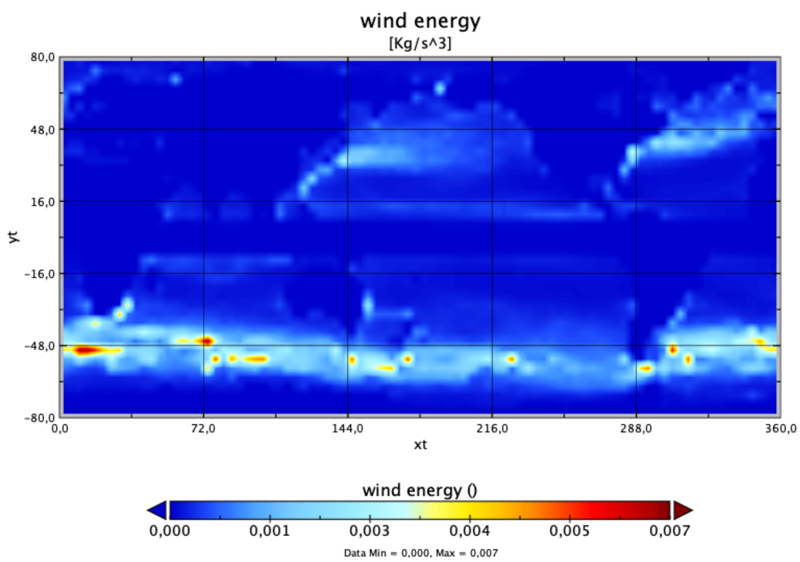
Figure: **Precomputed initial settings by** `Open_ITF_4deg.nc` **file:** (a): temperature, (b): salinity, (c): wind energy, (d): tidal energy, (e): zonal wind stress, (f): meridional wind stress and (g): bathymetry.



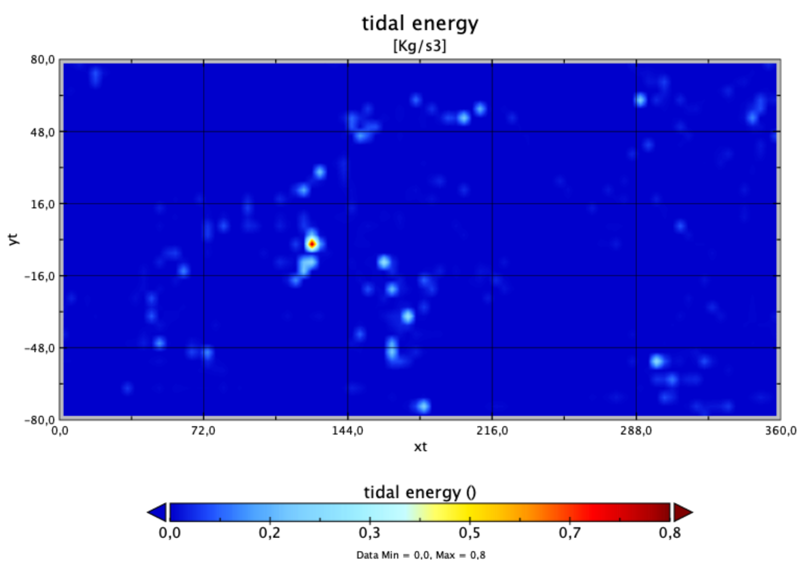
(a)



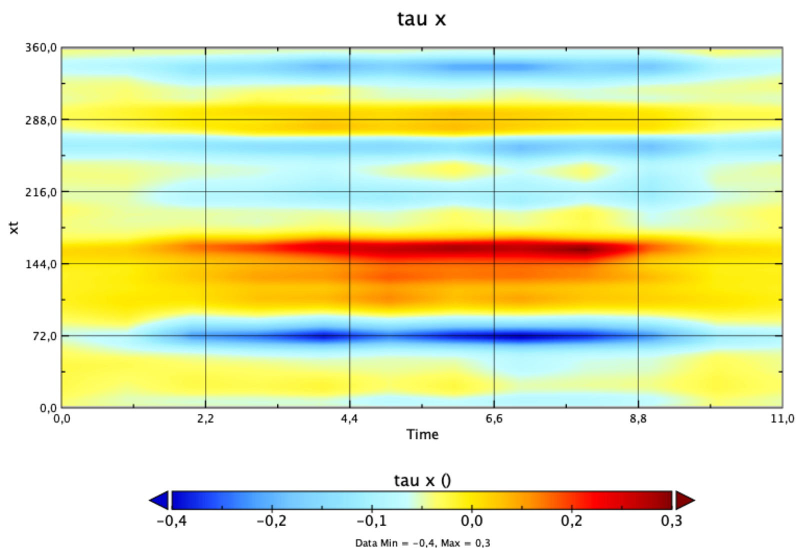
(b)


wind energy
[Kg/s^3]
80,0
48,0
16,0
-16,0
-48,0
-80,0
yt
0,0
72,0
144,0
216,0
288,0
360,0
xt
wind energy ()
0,000
0,001
0,003
0,004
0,005
0,007
Data Min = 0,000, Max = 0,007

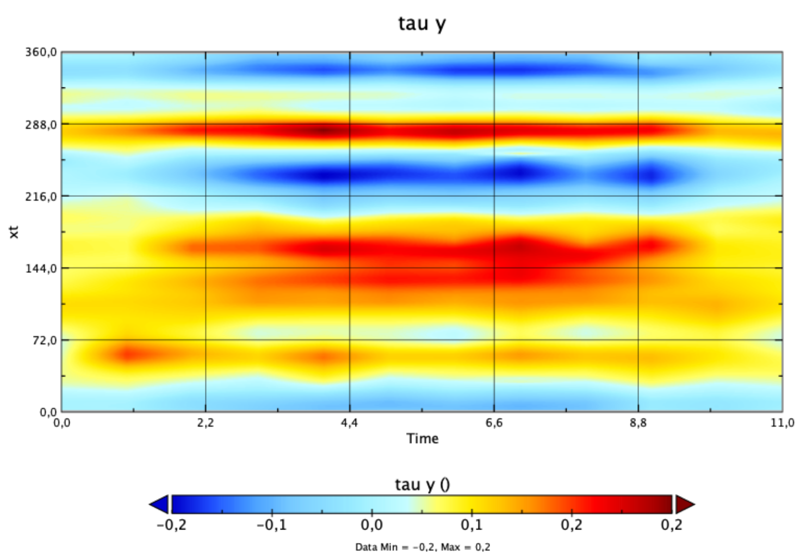
(c)


tidal energy
[Kg/s3]
80,0
48,0
16,0
-16,0
-48,0
-80,0
yt
0,0
72,0
144,0
216,0
288,0
360,0
xt
tidal energy ()
0,0
0,2
0,3
0,5
0,7
0,8
Data Min = 0,0, Max = 0,8

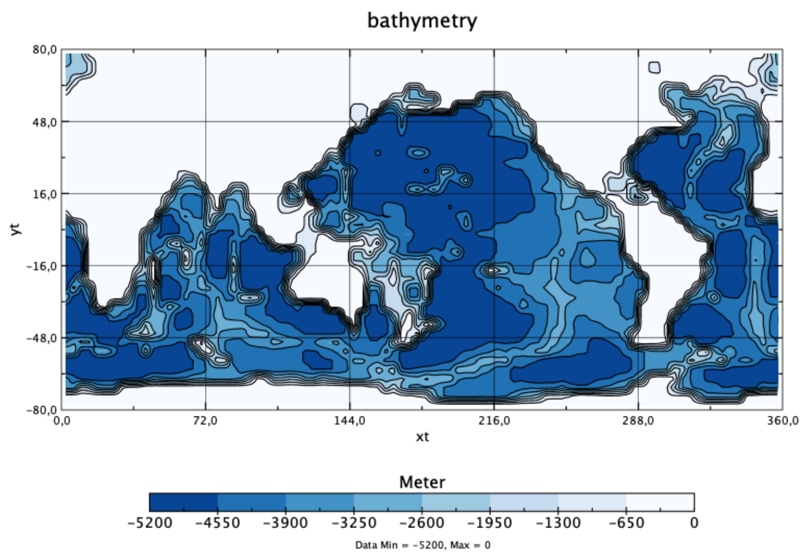
(d)


tau x
360,0
288,0
216,0
144,0
72,0
0,0
xt
0,0
2,2
4,4
6,6
8,8
11,0
Time
tau x ()
−0,4
−0,2
−0,1
0,0
0,2
0,3
Data Min = −0,4, Max = 0,3

(e)


tau y
360,0
288,0
216,0
144,0
72,0
0,0
xt
0,0
2,2
4,4
6,6
8,8
11,0
Time
tau y ()
−0,2
−0,1
0,0
0,1
0,2
0,2
Data Min = −0,2, Max = 0,2

(f)


bathymetry
80,0
48,0
16,0
-16,0
-48,0
-80,0
yt
0,0
72,0
144,0
216,0
288,0
360,0
xt
Meter
-5200
-4550
-3900
-3250
-2600
-1950
-1300
-650
0
Data Min = -5200, Max = 0

(g)